Compressing over-the-counter markets*

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Abstract

Over-the-counter markets are at the center of the global reform of the financial system. We show how the size and structure of these markets can undergo rapid and extensive changes when participants engage in portfolio compression, an optimisation technology that exploits multilateral netting opportunities. We find that tightly-knit and concentrated trading structures, as featured by many large over-the-counter markets, are especially susceptible to reductions of notional amounts and network reconfigurations resulting from compression activities. Using a unique transaction-level dataset on credit-default-swaps markets, we estimate reduction levels suggesting that the adoption of this technology can account for a large share of the historical development observed in these markets after the Global Financial Crisis. Finally, we test the effect of a mandate to centrally clear over-the-counter markets in terms of size and structure. When participants engage in both central clearing and portfolio compression with the clearinghouse, we find large netting failures if clearinghouses proliferate. Allowing for compression across clearinghouses by-and-large offsets this adverse effect.

Keywords: over-the-counter trading, multilateral netting, derivatives, networks, financial regulation

JEL codes: G20, G28, G15, C61, L14

1 Introduction

Over-the-counter (OTC) markets held a central role during the Global Financial Crisis (GFC). As a result, several jurisdictions mandated major regulatory reforms including central clearing, increased capital requirements and enhanced trading transparency. Underpinning these initiatives was the need to curb counterparty risk stemming from excessive leverage and the lack of transparency in the mutual positions of financial institutions.[1]

OTC markets are characterised by their large aggregate size in terms of notional obligations, to the level of hundreds of trillion of dollars. A most notable effect of the regulatory reforms was the downsizing of several OTC markets. For example, the market for Credit Default Swaps (CDS) featured a remarkable reduction in size - from USD 61.2 trillion outstanding at end-2007 to USD 8.3 trillion outstanding at mid 2018.[2] In principle, this 86% reduction could be seen as a mere reflection of lower trading activity. However, several sources have instead attributed its origin to the global adoption of a post trade risk management technique prompted by the new regulation, namely portfolio compression (see for example Vause (2010); Schrimpf (2015); ISDA (2015); Aldasoro and Ehlers (2018)) .

Portfolio compression is a multilateral netting technique which enables market participants to coordinate the replacement of existing contracts in order to reduce the size of their mutual obligations - thereby reducing counterparty risk - while maintaining the same underlying market risks. This technology relies on the solution of a convex optimisation problem on the network of outstanding obligations where constraints are set by participants themselves in line with their individual preferences. Figure 1 illustrates the process in a stylized example.

In the aftermath of the crisis, regulators recognized the need to limit excessive gross exposures (Cecchetti et al., 2009). As a result, new policies have generally supported the adoption of portfolio compression as a mean to reduce counterparty risk (see Section B for more details on the institutional background). More importantly, the increased levels of capital and margin requirements brought by the post-crisis regulation have also indirectly accelerated the private

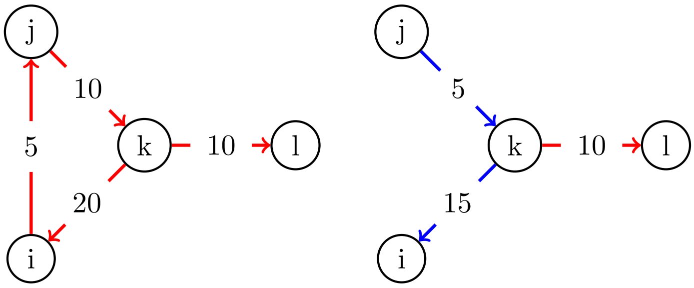
[1]Cases such as the American International Group (AIG) have illustrated how the opacity of OTC markets generates a counterparty risk externality. Acharya and Bisin (2014) show that this risk spills over from bilateral interdependencies and prevents the establishment of contracts with an adequately priced default risk premium. This externality, in turn, incentivises market participants to take on short positions with inefficiently large default risk. In general, it is too costly or infeasible in many realistic OTC market settings to fully internalize counterparty risks because it requires market participants to have the full information of the position of their counterparties.

[2]See the Bank of International Settlement Statistics on OTC derivatives outstanding: `https://www.bis.org/statistics/derstats.htm`

demand for portfolio compression. In fact, by reducing gross positions while maintaining net balances unchanged, this technology allows market participants to reduce both requirements without affecting their market risk (Duffie, 2017). The leading compression service provider TriOptima reports over one quadrillion USD in notional elimination through their service (see Section B). Albeit the impact, analytical and empirical analysis have been so far limited due to the lack of adequate data and the opacity of the practice.

In this paper, we present an analytical framework that explains how the size of OTC markets can be subject to large and rapid reductions when participants engage in portfolio compression cycles. In essence, our approach studies the level of feasibility and efficiency of a compression operation by mapping the post-trade optimisation problem into a min-cost flow problem applied to the network of outstanding obligations. First, the feasibility analysis characterizes the conditions under which a compression operation can strictly reduce total notional. The efficiency analysis then evaluates the maximum theoretical level of notional that a compression operation can eliminate. We apply our framework to a unique transaction-level dataset and estimate reduction ranges at par with the historical levels exhibited by the CDS markets after the GFC. In addition, we study the interplay between central clearing - another major regulatory reform - and portfolio compression. We find large netting failures when clearinghouses proliferate and show that multilateral compression across clearinghouses can by-and-large compensate this adverse effect.

In our model, netting opportunities exist when at least one participant intermediates a set of fungible obligations. The total amount of notional eligible for compression, henceforth market *excess*, is further determined by the existence and length of chains of intermediation in the market. The exact fraction of excess that can be compressed is bounded by individual portfolio preferences and regulatory constraints. We study a spectrum of benchmark preference settings by investigating their feasibility and efficiency as well as by providing a structural characterisation of their optimal solutions. These benchmarks differ in the extent to which participants accept reconfigurations of their original sets of counterparties. For instance, dealer banks may be indifferent vis-à-vis changes in their trades with other dealers while being conservative on the trading relationships they have established with their customers. We derive an ordering of the full spectrum of preferences, highlighting a trade-off between the efficiency of a compression cycle and the degree of tolerances set by portfolio preferences: higher netting opportunities arise



(a) Before Compression (b) After Compression

Figure 1: A graphical example of portfolio compression. Panel (a) exhibits a market consisting of 4 market participants (i, j, k, l) with short and long positions on the same asset with different notional values. The aggregate gross notional of the market is 45. Panel (b) shows a possible compression solution to the market: by eliminating the obligations between i, j and k and generating two new obligations, the net position of each participant is unchanged while the gross positions of i, j and k have been reduced by 5. In aggregate, market size has been reduced by 15 units.

when participants are less conservative in their original sets of counterparties.

Next, we empirically estimate the levels of excess and compression efficiency using a unique transaction-level dataset. To the best of our knowledge, this is the first calibrated analysis of the potential impact of a market-wide adoption of portfolio compression. We use a dataset consisting of all CDS contracts bought and sold by legal entities based in the European Union (EU) and all their counterparties. First, we find that the majority of markets exhibit levels of excess accounting for 75% or more of their total notional size. Furthermore, even the most conservative compression scenario, in which all participants preserve their original trading relationships, eliminates on average more than 85% of the excess in markets, for a total of at least two thirds of their original size.[3]

These results are explained by the observed tightly-knit structure of the intra-dealer segment which allows for large excess elimination while preserving counterparty trading relationships. Nevertheless, we find that the efficiency of a conservative compression is impaired if market participants seek to bilaterally net out their positions beforehand. This effect is dampened when compression preferences are relaxed in the intra-dealer segment.

Finally, we run a stylized study of market excess and compression efficiency when participants adopt both portfolio compression and central clearing. Despite the multilateral netting opportunities brought by centralization, clearing also duplicates the notional value of each obligations. The effect of central clearing on market excess is therefore ambiguous by construction,

[3]These results and statistics are in line with evidence provided by several reports. See for example Vause (2010) for CDS globally and OCC (2016) for US derivatives.

especially when multiple clearinghouses exist. When clearing takes place with one single central clearing counterparty (CCP), our calibrated results indicate that this setting is dominated by - but close to - multilateral compression without central clearing. A proliferation of CCPs significantly and systematically increases the gap. Markets with several CCPs prevent large netting opportunities among common clearing members. Remarkably, we find that such effects are by-and-large reduced when a compression mechanism exist across CCPs, that is, when members of several clearinghouses can compress beyond their bilateral exposure to each clearinghouse independently. We conclude with a discussion on the systemic implications of our results for liquidity and financial stability.

1.1 Literature review and contribution

The results of this paper contribute to several strands of the literature as well as ongoing policy debates.

Empirical studies including Shachar (2012); Benos et al. (2013); Peltonen et al. (2014); Ali et al. (2016); D'Errico et al. (2017); Abad et al. (2016) show that OTC markets are characterized by large concentration of notional within the intra-dealer segment. In particular, D'Errico et al. (2017) observe that in the global CDS market, intermediaries form a strongly connected structure which entails several closed chains of intermediation. The authors also show that between 70% and 80% of the notional in CDS markets is in the intra-dealer market across all reference entities. Atkeson et al. (2013) report that, in the US, on average, about 95% of OTC derivatives gross notional is concentrated in the top five banks. Abad et al. (2016) report similar levels for interest rate swap, CDS and foreign exchange markets in the EU segment. This paper contributes by proposing a well defined measure of the market-level gap between gross and net notional. This so-called excess indicator, in turn, corresponds to the maximum amount of notional eligible for compression. Importantly, our results show that an explicit modeling of the entire network of bilateral obligations is necessary to estimate the efficiency of portfolio compression. We find that it is the combination of high notional concentration and dense cycles of intermediation that allows for large compression of excess even under conservative preferences.

Theoretical analyses of OTC markets have addressed trading frictions and prices with a focus on the role of intermediaries (see Duffie et al., 2005; Lagos and Rocheteau, 2009; Gofman, 2016; Babus and Hu, 2017). In particular, Atkeson et al. (2015) and Babus et al. (2018) find equilibrium conditions to observe large concentration in few market participants. On the one hand, Babus et al. (2018) show the importance of the centrality of a dealer to reduce trading costs. On the other hand Atkeson et al. (2015) show that only large participants can enter the market as dealers in order to make intermediation profits. Other studies have analyzed counterparty risk pricing in OTC markets. Acharya and Bisin (2014) and Frei et al. (2017) show that opacity and trade size limits usually prevent efficient risk pricing. While, in this paper, we study arbitrary sets of trading relationships, our results show that, under realistic assumptions, the adoption of post-trade technologies can largely impact the size of dealers - and the market as a whole - thus making OTC markets prone to rapid structural reconfigurations when participants coordinate. This result is particularly relevant in lights of the role played by large and mispriced positions held by OTC dealers during the GFC as discussed by Cecchetti et al. (2009).

The study of post-trade services has so far mainly focused on the costs and benefits of central clearing. Duffie and Zhu (2011) provide the ground work of this strand of research. The authors show that, while central clearing helps to reduce exposures at the asset class level, clearing heterogeneous asset classes removes the benefits of netting. Cont and Kokholm (2014) show that a more risk sensitive approach to asset classes can alleviate the need to concentrate all netting activities in one single CCP. Duffie et al. (2015), Glasserman et al. (2015) and Ghamami and Glasserman (2017) study the impact of clearing on collateral and capital requirements and show that trading and liquidation costs can be higher or lower depending on the proliferation of CCPs and the extent to which netting opportunities can be exploited. In addition, Amini et al. (2016) show that netting inefficiencies resulting from partial clearing may be more detrimental - in terms of bank shortfall - than no netting at all. The results of this paper on the effect of multiple CCPs provide a quantitative assessment of the loss in netting efficiencies and its impact on market excess. Furthermore, the finding that compression across CCPs vastly removes netting inefficiencies shows that multilateral compression among CCPs can address the trade-off between full centralization and efficiency losses introduced by Duffie and Zhu (2011).

Regarding the theory of portfolio compression, O'Kane (2017) stands as the main theoretical contribution. The author numerically analyzes the performances of different versions of compression algorithm on a synthetic network where all banks are connected. The author shows that, if performed optimally, compression mitigates counterparty risk. Our work differs in several ways. First, we study sparse and concentrated market structures which correspond to a realistic setting distinguishing dealers from customers. In addition, we derive analytical solutions to the efficiency ranking of compression solutions as a function of a spectrum of portfolio preferences. Finally, we apply our framework to transaction-level data and identify bounds of compression for each preference setting in OTC derivatives markets.

Finally, our work relates to the growing stream of literature highlighting the important relationship between financial interconnectedness, stability and policy making (see Allen and Babus, 2009; Yellen, 2013). These works explore the role of interdependencies on the propagation of distress (Allen and Gale, 2000; Elliott et al., 2014; Acemoglu et al., 2015) and regulatory oversight (Alvarez and Barlevy, 2015; Roukny et al., 2016; Erol and Ordoñez, 2017; Bernard et al., 2017). Our paper shows how post-trade optimisation can affect the network of outstanding positions in financial markets. This matters both for the stability of such markets and for the tools required by policy makers to address market stability. Compression reconfigures counterparty risk and intermediation chains which have held a central role in the propagation of distress during the 2007-2009 financial crises (Haldane, 2009; European Central Bank, 2009).

1.2 Paper organisation

The rest of the paper is organized as follows. In Section 2, we present our stylized model of OTC market and the analysis of market excess. Section 3 presents a mathematical definition of portfolio compression; introduces benchmark preference settings; identifies feasibility and efficiency levels of each approach. In Section 4 and 5, we report the results of our empirical analysis of excess and compression efficiency in real OTC derivatives markets. In Section 6, we complement our framework with the addition of central clearing and study the impact on market excess. Last, we conclude and discuss implications of our results. The appendices provide proofs of the propositions and lemmas, an overview of the institutional background, additional results as well as the analytical details for the algorithms used in the paper.

2 The model

We consider an over-the-counter (OTC) market composed of n market participants denoted by the set $N = \{1, 2, ..., n\}$. These participants trade contracts with each other and establish a series of bilateral positions resulting in outstanding gross exposures represented by the $n \times n$ matrix $\mathbf{E}$ with nonnegative real elements e_{ij} and a zero diagonal, $e_{ii} = 0$ for all $i \in N$. By convention, the direction is from the seller i to the buyer j with $i, j \in N \times N$. While we keep the contract type general, we assume that the resulting obligations are fungible: they have the same payoff structure from the market participants' perspective and can therefore be algebraically summed. The whole set of outstanding obligations in the market constitutes a financial *network* or *graph* $G = (N, \mathbf{E})$.

The gross position of a market participant i is the sum of all obligations' notional value involving her on either side (i.e., buyer and seller): $v_i^{\text{gross}} = \sum_j e_{ij} + \sum_j e_{ji} = \sum_j (e_{ij} + e_{ji})$. The net position of i is then the difference between the aggregated sides: $v_i^{\text{net}} = \sum_j e_{ij} - \sum_j e_{ji} = \sum_j (e_{ij} - e_{ji})$. The *total gross notional* of the whole system is the sum of the notional amounts of all obligations: $x = \sum_i \sum_j e_{ij}$.

Finally, market participants can either be customers or dealers. Customers only enter the market to buy or sell a given contract: if $i \in N$ is a customer then $\left(\sum_{j \in N} e_{ij}\right) \cdot \left(\sum_{j \in N} e_{ji}\right) = 0$. In contrast, dealers also intermediate between other market participants: if $i \in N$ is a dealer then $\left(\sum_{j \in N} e_{ij}\right) \cdot \left(\sum_{j \in N} e_{ji}\right) > 0$. We therefore map the network into three types of trading relationships in the market: dealer-customer, dealer-dealer and customer-customer.

2.1 Market excess

Figure 2 shows the network of obligations of an actual OTC market of CDS contracts. Customers buying the CDS contract are on the left hand-side (green), customers selling the CDS contract are on the right hand-side and dealers are in the middle (blue and purple where purple nodes are the G-16 dealers). While buyers and sellers have a combined gross share of less than 5%, their net position is equal to their gross position. In contrast, the set of dealers covers more than 95% of gross market share while, on average, only one fifth is covered by net positions. As a result, 76% of the notional held by dealers is the result of offsetting positions. Offsetting positions constitute the netting set of interest for portfolio compression. Below we formalise the intuition that stands behind the identification of netting opportunities at the market level.

Consider a post-trade optimisation process that takes the market represented by the network $G = (N, \mathbf{E})$ and returns a network $G' = (N, \mathbf{E}')$ where the aggregate notional amount is minimised subject to several constraints. In the context of portfolio compression, we focus on a

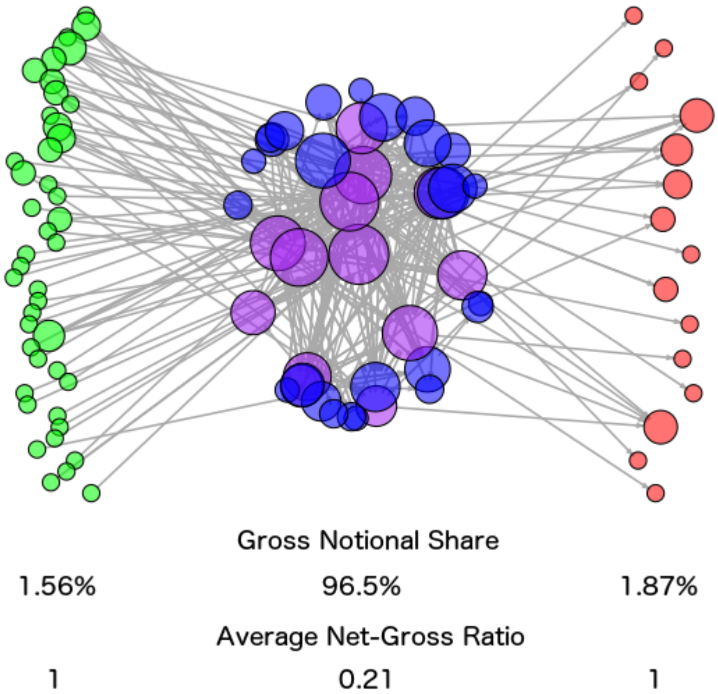


Figure 2: Network illustration of an OTC derivatives market, which maps all outstanding obligations for CDS contracts written on the same reference entity for the month of April 2016. The data were collected under the EMIR reporting framework and thus contain all trades where at least one counterparty is legally based in the EU. Green nodes correspond to buyers of the contract. Red nodes correspond to sellers of the contract. Purple nodes are G-16 dealers. Blue nodes are dealers not belonging to the G-16 dealer set. The first line below the network reports the share of gross notional based on individual positions for the segments: buyers, dealers, sellers. The second line reports the average net-gross ratio for each segment.

net-equivalence constraint which maintains the net position of each institution before and after the operation: $v_i^{net} = v_i'^{net}, \forall i \in N$. Under this constraint, it is possible to derive the minimum level of gross notional of the returned market G'. This value corresponds to the original net out-flow of G.

Proposition 1. *Given a market $G = (N, \mathbf{E})$, the minimum notional amount that a net-equivalent market G' can exhibit is the net out-flow given by:*

$$x'(G) = \frac{1}{2}\sum_{i=1}^{n} |v_i^{net}| = \sum_{i:v_i^{net}>0}^{n} v_i^{net} = -\sum_{i:\, v_i^{net}<0}^{n} v_i^{net}. \tag{1}$$

Proof. Proof see Appendix A. ■

Absent any additional condition, there always exists a reconfiguration of trades that produces a notional amount such that it corresponds to half of the aggregated absolute original net positions. This level is, in turn, the minimum notional amount that a net-equivalent operation can produce. Using Proposition 1, we define the market excess as the difference between the aggregate gross notional exhibited by the market and the minimum notional attainable by a net-equivalent market (as per Equation 1). By construction, the excess in the market is the amount of notional generated by obligations that offset each other. Section 3 studies the conditions under which the minimum notional can be attained.

Definition (Excess)**.** *The excess in the market is defined as*

$$\begin{aligned}\Delta(G) &= x - x'(G) \\ &= \left(\sum_{i=1}^{n}\sum_{j=1}^{n} e_{ij} - \frac{1}{2}\sum_{i=1}^{n} |v_i^{net}|\right)\end{aligned} \tag{2}$$

From Definition 2.1, we observe that the excess in a market is strictly positive if at least one participant i exhibits a gross position larger than her net position, i.e. when the participant is a dealer.

Lemma 1. *Given a market $G = (N, E)$, market excess is strictly positive if and only if at least one participant is a dealer. Dealer enabled over-the-counter markets always exhibit strictly positive market excess.*

Proof. Proof see Appendix A. ■

This result relies on the effect of intermediation over net-to-gross ratios: as long as a participant holds claims in opposite directions, her gross position is larger than her absolute net position. This, in turn, results in strictly positive market excess. The result also explicitly shows why the existence of notional excess is intrinsic to OTC markets as these markets are characterised by dealer intermediated trades. [4] Note that even if some OTC markets exhibit customer-customer trading relationships, those interactions do not contribute to notional excess. We document these features in the empirical Section of this paper.

2.2 Market excess decomposition

Given the existence of multiple types of trading relationships (e.g. dealer-customer trading, intra-dealer trading.), we analyse the effect of a market segmentation on the measurement of market excess. Markets can be decomposed into the intra-dealer segment and the customer segment, respectively. The intra-dealer (sub-)market only contains obligations between dealers while the customer (sub-)market contains obligations where at least one counterparty is a customer. Formally we have:

Definition (Intra-dealer and customer market). *We partition the matrix of obligations* $\mathbf{E}$ *into two complementary matrices:* $\mathbf{E^D}$ *where* $e^D_{ij} = e_{ij}$ *if both* i *and* j *are dealers and zero otherwise;* $\mathbf{E^C}$ *where* $e^C_{ij} = e_{ij}$ *if at least* i *or* j *is a customer and zero otherwise. We have:* $\mathbf{E} = \mathbf{E^D} + \mathbf{E^C}$.

We find that, in general, the excess is super-additive and cannot be linearly decomposed:

Proposition 2. *Given a market* $G = (N, \mathbf{E})$ *and the following partition* $\mathbf{E} = \mathbf{E^1} + \mathbf{E^2}$ *such that* $G^1 = (N, \mathbf{E^1})$ *and* $G^2 = (N, \mathbf{E^2})$*, then:*

$$\Delta(G) \geq \Delta(G^1) + \Delta(G^2)$$

Furthermore, applying the dealer-customer partitioning such that $\mathbf{E} = \mathbf{E^C} + \mathbf{E^D}$*, we have that*

$$\Delta(N, \mathbf{E}) = \Delta(N, \mathbf{E^D}) + \Delta(N, \mathbf{E^C})$$

[4] Note the special case of bilaterally netted positions. In the business practice of some instruments such as CDS contracts, two institutions sometimes terminate or reduce their outstanding bilateral position by creating an offsetting position (i.e., obligation of similar characteristics in the opposite direction). Such setting also generates excess. While this mechanism cannot be framed as intermediation per se, our formal network definition still applies. From a purely mathematical perspective, both participants are active on the buy and sell side and the related results remain.

when

1. $\sum_{h \in D}(e_{dh} - e_{hd}) = 0, \quad \forall d \in D$

   or

2. $\sum_{c^+ \in C^+} e_{c^+d} - \sum_{c^- \in C^-} e_{dc^-} = 0, \quad \forall d \in D$

Where D is the set of dealers and the set C^+ (resp. C^-) includes all customers with positive (resp. negative) net positions: $\{i|v_i^{net} > 0$ and $i \in C\}$ (resp. $\{i|v_i^{net} < 0$ and $i \in C\}$) with C being the set of all customers.

Proof. Proof see Appendix A. ■

This result implies that strict additivity, $\Delta(N, \mathbf{E}) = \Delta(N, \mathbf{E^D}) + \Delta(N, \mathbf{E^C})$, only holds when all dealers jointly have a zero net position in at least one of the two market segments.

3 Portfolio compression

Building on the framework introduced in the previous section, we now study how participants can coordinate to eliminate offsetting obligations using portfolio compression. For sake of simplicity, we do not explicitly model the incentives for participants to compress (see Section B). Rather, we focus on the optimisation formulation of the problem as well as the feasibility and efficiency conditions. Therefore, portfolio preferences are considered exogenous at this stage. For each set of preferences, we identify when and how much excess can be eliminated. An analysis of endogenously driven equilibria is left for future research.

In its minimal form, portfolio compression can be represented as a compression operator $c : G \to G'$, where $G' = (N, \mathbf{E}')$ and $\mathbf{E}'$ is the solution of a linear program which minimizes aggregate gross notional subject to prior net-positions. From a network perspective, this problem is analogous to a min-cost flow problem. [5]

A direct corollary of Lemma 1 is that participants can effectively engage in portfolio compression if the market exhibits intermediation.

Corollary 1. *Compression can only take place if there is at least one dealer in the market.*

[5] We chose to focus the objective function of the compression problem on notional minimisation to reflect the current industry practice which has been confirmed through interactions with the largest compression providers. Future development may also include other aspects such as collateral requirement optimisation.

However, compression problems often include additional constraints on top of the net-equivalence condition such as participants' individual preferences and regulatory policies. For instance, market participants may be unwilling to compress some specific bilateral positions; policy-makers may prevent specific obligations from being established. These channels lead to multiple constraints on each pair of participants. Therefore, the above corollary constitute a necessary but not sufficient condition. The sufficiency condition must be expressed as a function of all applicable constraints.

For each possible bilateral obligation between i and j, we consider the most binding constraint and use it as an additional condition in the program. We refer to this selected set of constraints as *compression tolerances.*

Formally, compression tolerances form a set of bilateral bounds in the following way:

Definition (Compression tolerances)**.** *Given an original market* $G = (N, \mathbf{E})$*, a compression operator* $c : G \to G'$ *such that* $G' = (N, \mathbf{E}') = c(G)$ *is said to satisfy the set of compression tolerances* $\Gamma = \{(a_{ij}, b_{ij}) | a_{ij}, b_{ij} \in \mathbb{R}^+, i, j \in N\}$ *if*

$$a_{ij} \leq e'_{ij} \leq b_{ij}$$

with $0 \leq a_{ij} \leq e_{ij}$, $e_{ij} \leq b_{ij}$.

Including tolerances in the program, we obtain a general formulation of the optimization problem. Let $G = (N, \mathbf{E})$ be the original market; Γ be the set of all compression tolerances; $\mathbf{E}'$ be the matrix of exposures after compression. A compression operator $c : G \to G'$ where $G' = (N, \mathbf{E}')$ solves the compression problem by finding the optimal exposure matrix $\mathbf{E}'$ according the following program:

Problem 1 (General compression problem)**.**

$$\begin{array}{lll}
\min & \sum_{i,j} e'_{ij} & \\
s.t. & \sum_j \left(e'_{ij} - e'_{ji} \right) = v_i^{net} & \forall i \in N \quad \textit{[net-equivalence constraint]} \\
 & a_{ij} \leq e'_{ij} \leq b_{ij} & \forall i, j \in N \times N \quad \textit{[compression tolerances]}
\end{array}$$

with $a_{ij} \in [0, \infty)$, $b_{ij} \in [0, \infty)$ and $a_{ij} \leq b_{ij}$.

The set of all individual compression tolerances determines the exact amount of offsetting obligations to be included in the compression exercise. We refer to the uncompressed excess as the *residual excess* which corresponds to $\Delta(c(G)) = \Delta(G')$.

3.1 Counterparty preference settings

In the following, we consider a general spectrum of preferences based on counterparty relationships. We start with two benchmark settings. In the first setting, participants are *conservative*: they only allow for reductions of established obligations. In the second setting, participants are indifferent vis-à-vis changes in their trading relationships. These settings correspond to the following sets of compression tolerances, $\{(a_{ij}, b_{ij}) = (0, e_{ij}) \;\; \forall (i,j) \in N^2\}$ and $\{(a_{ij}, b_{ij}) = (0, +\infty) \;\; \forall (i,j) \in N^2\}$, respectively.

We refer to the first setting as *conservative* and to the second as *non-conservative*. Intuitively, the non-conservative case provides the highest levels of compression tolerances: it discards all counterparty constraints. In the conservative case: compression tolerances are such that $e'_{ij} \leq e_{ij}$ for all bilateral positions. Hence, all participants are willing to reduce or eliminate their original obligation but no new relationship can be introduced between participants not trading ex-ante. We then consider two additional settings: a hybrid combination of conservative and non-conservative tolerance distinguishing between dealer-customer and intra-dealer trades respectively, and a bilateral compression setting.

Non-conservative compression

In the non-conservative compression setting, only the net-equivalence constraint binds. The mathematical formulation is thus given by the following problem:

Problem 2 (Non-conservative compression problem)**.**

$$
\begin{aligned}
\min \quad & \textstyle\sum_{i,j} e'_{ij} \\
s.t. \quad & \textstyle\sum_{j} \left(e'_{ij} - e'_{ji} \right) = v_i^{net} \quad \forall i \in N \;\; \textit{[net-equivalence constraint]} \\
& e'_{ij} \in [0, \infty) \quad \forall i,j \in N \times N \quad \textit{[non-conservative compression tolerances]}
\end{aligned}
$$

Individual gross positions are not bounded upward, therefore, compression tolerances are set to $a_{ij} = 0$ and $b_{ij} = \infty, \forall (a_{ij}, b_{ij}) \in \Gamma$ where Γ is the set of all compression tolerances. In practice, such setting is unlikely to be the default modus operandi. However, it is conceptually useful to study as it sets the benchmark for the most tolerant setting.

Conservative compression

In the conservative compression setting, the matrix of obligations in the compressed market is strictly obtained from the original matrix of bilateral positions. Formally, we have:

Problem 3 (Conservative compression problem)**.**

$$
\begin{array}{lllr}
\min & \sum_{ij} e'_{ij} & & \\
s.t. & \sum_j \left(e'_{ij} - e'_{ji}\right) = v_i^{net} & \forall i \in N & \textit{[net-equivalence constraint]} \\
 & 0 \leq e'_{ij} \leq e_{ij} & \forall i, j \in N \times N & \textit{[conservative compression tolerances]}
\end{array}
$$

The resulting graph $G' = (N, \mathbf{E}')$ is a sub-graph of the original market $G = (N, \mathbf{E})$. Such setting is arguably close to the way most compression cycles take place in derivatives markets.[6] To illustrate the implementation of both approaches, we provide a simple example of a market with 3 market participants in Appendix C.

Hybrid compression

In many realistic settings, compression tolerances can be subject to the economic role of specific trading relationships. In the following, we consider a set of participants' preferences that combines properties from these two benchmarks.

Assumption 1. *Dealers prefer to keep their intermediary role with customers.*

Assumption 2. *Dealers are indifferent vis-à-vis their bilateral positions with other dealers. Intra-dealer obligations can be switched at negligible cost.*

The first assumption states that dealers value their role with customers. They will reject any compression solution that affects their bilateral positions with customers. Therefore, dealers set low compression tolerances on their customer related obligations. The second assumption posits that the intra-dealer network forms a club in which instances of a specific obligation do not signal a preference towards a given dealer counterparty. As a result, switching counterparties in the intra-dealer network has negligible costs in comparison with the overall benefits of compression. Therefore, dealers set high compression tolerances in the intra-dealer segment.

Using Definition 2.2, we have the following formal problem definition:

[6] We thank Per Sjöberg, founder and former CEO of TriOptima, for fruitful discussion on these particular points.

Problem 4 (Hybrid compression problem)**.**

$$
\begin{array}{lll}
\min & \sum_{ij} e'_{ij} & \\
s.t. & \sum_j \left(e'_{ij} - e'_{ji} \right) = v_i^{net} \quad \forall i \in N & \textit{[net-equivalence constraint]} \\
 & e'_{ij} = e^C_{ij} \quad \textit{if} \quad i \textit{ or } j \textit{ is a customer} & \textit{[customer segment]} \\
 & e'_{ij} \in [0, \infty) \quad \textit{if} \;\; i \textit{ and } j \textit{ are dealers} & \textit{[intra-dealer segment]}
\end{array}
$$

.

Bilateral compression

Finally, we study a simple preference setting: bilateral compression. In this case, market participants do not exploit multilateral netting opportunities. Formalizing this compression approach allows us, in part, to assess the added-value of a third party compression service provider when comparing efficiencies between bilateral and multilateral compression solutions. In our framework, bilateral compression is defined as follows: for each pair of market participants i and j, we have $e'_{ij} = \max\{e_{ij} - e_{ji}, 0\}$ and for each market participant i we have $\sum_j \left(e'_{ij} - e'_{ji} \right) = v_i^{net}$.

3.2 Compression feasibility and efficiency

For each setting, we identify both feasibility conditions and efficiency levels. We study the feasibility of a compression setting by identifying the conditions under which the operation strictly reduces excess. We study the efficiency of a compression setting by characterizing how much excess can be optimally eliminated subject to the associated set of compression tolerances. The results show the existence of a trade-off between the degree of portfolio conservation and the level of efficiency.

Non-conservative compression

Under a non-conservative compression setting, the original bilateral positions do not constrain the outcome, only the net and gross positions of each participant do. We can thus generalize Corollary 1 to reach the feasibility condition as follows:

Proposition 3. *Given a market $G = (N, \mathbf{E})$ and compression operator $c^{nc}()$ solving the non-conservative compression problem, the amount of eliminated excess is strictly positive if and only if there is at least one dealer in the market.*

Proof. Proof see Appendix A. ■

Any compression problem with a non-conservative set of tolerances is feasible if the market exhibits intermediation. In terms of efficiency, we obtain the following result:

Proposition 4. *Given a market $G = (N, \mathbf{E})$, there exists a set of non-conservative compression operators C^{nc} such that all the excess is eliminated. Moreover, let $G' = c^{nc}(G)$ such that $c^{nc}()$ is an operator from C^{nc}, then G' has a bi-partite structure.*

Proof. Proof see Appendix A. ■

Non-conservative compression eliminates all the excess in a market. However, non-conservative solutions may achieve this result through different network arrangements. Let C^{cn} be the set of all such compression operators and c^{cn} be an operator from this set. Then, naturally, the common structural feature to all markets resulting from $c^{nc}(G)$ is that they exhibit a bi-partite structure. The proof of existence stems from the following generic algorithm: from the original market, compute all the net positions then empty the network of obligations and arbitrarily generate obligations such that the gross and net positions are equal. As net and gross positions are equal, the resulting market does not exhibit any intermediation. Recall from Lemma 1 that if all intermediation chains are broken, the market exhibits no excess. We also obtain the following corollary:

Corollary 2. *Given a market $G = (N, \mathbf{E})$ and a compression operation $c()$ solving the general compression problem, the residual excess is zero if and only if there is no more intermediation in the compressed market.*

The resulting market is characterized by a bi-partite structure where participants are strictly associated with the buying or selling customer set. For illustrative purposes, we provide a simple algorithm for this compression setting in Appendix E.

Conservative compression

When compression tolerances are conservative, the compression operator can only treat offsetting obligations. In contrast with the non-conservative case, conservative compression cannot be applied to general chains of intermediation. Below we show that, only when directed chains of intermediation are closed, can conservative compression take place.

Let us first formalize the concept of directed closed intermediation chains:

Definition (Directed Closed Chain of Intermediation)**.** *Given a graph $G = (N, \mathbf{E})$, a directed closed chain of intermediation is a set of strictly positive obligations such that each node is visited strictly twice.*

This structure constitutes the necessary and sufficient condition for conservative compression to be feasible in a market:

Proposition 5. *Given a market $G = (N, \mathbf{E})$ and a compression operator $c^c()$ solving the conservative compression problem, the amount of eliminated excess is strictly positive if and only if there is at least one directed closed chain of intermediation in the market.*

Proof. Proof see Appendix A. ∎

In contrast with the non-conservative approach, the efficiency of conservative compression is determined by the underlying network structure. In the following, we analyze the efficiency of conservative compression when applied to a dealer-customer network structure as is empirically observed in OTC markets.

We start by showing that if the market exhibits a dealer-customer structure, conservative compression does not eliminate all the market excess (see Section 4 for further empirical evidence).

Proposition 6. *Given a market $G = (N, \mathbf{E})$ and a compression operator $c^c()$ solving the conservative compression problem, the residual excess is strictly positive if there is at least one dealer simultaneously buying from and selling to customers.*

Proof. Proof see Appendix A. ∎

When a dealer i intermediates between customers on both sides (i.e., $\sum_j e^C_{ij} > 0$ and $\sum_j e^C_{ji} > 0$), the resulting chains of intermediation are necessarily open. In turn, these chains cannot be conservatively compressed and the residual excess of the compression solution is strictly positive.

In the case of a single closed chain of intermediation, the optimal conservative solution is given by the following result:

Lemma 2. *Given a market consisting of one directed closed chain $G^K = (N, \mathbf{K})$, consider the set of optimal compression operators C solving the conservative compression problem, then the* *solution is given by*

$$e'_{ij} = e_{ij} - \varepsilon \quad \forall e'_{ij} \in \mathbf{K}'$$
$$\Delta(c^c(G^K)) = \Delta(G^K) - |K|\varepsilon \quad \forall c^c \in C$$

where $\varepsilon = \min_{e_{ij}}\{\mathbf{K}\}$*,* $|K|$ *is the total number of edges in the set* $\mathbf{K}$ *and* $\mathbf{K}'$ *is the resulting set of edges:* $c^c(G^K) = (N, \mathbf{K}')$

Proof. Proof see Appendix A. ■

Lemma 2 shows that, in a single directed closed chain, the optimal conservative solution consists in eliminating the obligation with the lowest notional value (i.e., ε) and accordingly adjusting all other obligations in the chain to maintain net-equivalence. The larger the length of the intermediation chain and the higher the minimum notional obligation value on the chain, the more excess can be eliminated conservatively.

When the original market exhibits several closed chains of intermediation, the exact arrangement of chains in the network is critical to determine the resulting efficiency. In Appendix D, we discuss cases of entangled chains (i.e., intermediation chains with common obligations) with different ordering effects. In general, it is not possible to determine the residual excess of a conservative compression without further assumptions on the underlying network structure. In order to guarantee a global solution, linear programming techniques such as the network simplex can be used. We elaborate more on this in Section 5.

We can however characterize the topological structure of the optimal solution. We obtain the following result for any conservative compression solution:

Proposition 7. *Given a market* $G = (N, \mathbf{E})$ *and a compression operator* $c^c()$ *solving the conservative compression problem, any solution* $G' = c^c(G)$ *is acyclic.*

All closed chains of intermediation can be conservatively compressed. As a result, the above Proposition states that all optimal solutions will be characterized by an acyclic topological structure. Note that, as our objective function is set on the amount of excess that is removed, multiple directed acyclic solutions can, in principle, coexist.

The results from Proposition 5, Lemma 2 and Proposition 7 show that the set of closed chains of intermediation present in a market determine the efficiency of a conservative compression. More specifically, the number of closed chains, their length and their minimum notional obligation constitute the positive factors that partially generate larger efficiency gains for a conservative compression. Markets exhibiting such features are usually referred to as markets with tightly-knit structures. Establishing the full extent of residual and redundant excess requires knowledge on the exact market network structure. Section 5 provides such analysis using transaction-level data.

Hybrid compression

The hybrid compression setting is a combination of (i) a non-conservative setting in the intra-dealer segment $\mathbf{E^D}$ and (ii) a conservative setting in the customer segment $\mathbf{E^C}$.

Corollary 3. *The feasibility conditions of the hybrid setting are*

- *non-conservative condition for* $\mathbf{E^D}$
- *conservative condition for* $\mathbf{E^C}$

Note that, in a dealer-customer market, a hybrid compression will only affect the intra-dealer segment because no closed chains of intermediation exists in the customer segment. As a result, the intra-dealer network will form a bipartite graph with zero residual intra-dealer excess.

Proposition 8. *Given a market $G = (N, \mathbf{E})$, if excess is linearly decomposable then, a compression operator $c^h()$ solving the hybrid compression problem produces the following residual excess:* $\Delta(c^h(N, \mathbf{E})) = \Delta(N, \mathbf{E^C})$

Proof. Proof see Appendix A. ∎

In case the excess is additive, the efficiency of hybrid compression is straightforward. In case it is not (see condition under Proposition 2), the situation is similar to the conservative setting: a specific algorithm (e.g., network simplex) must be implemented to obtain the exact level of efficiency.

Bilateral compression

In terms of feasibility, the mere existence of excess is not enough for bilateral compression to be applicable. In particular, we need at least two obligations between the same pair of counterparties and of opposite direction. Formally, we have the following results:

Proposition 9. *Given a market $G = (N, \mathbf{E})$ and a compression operator $c^b()$ solving the bilateral compression problem, the total amount of excess eliminated is strictly positive if and only if there are at least two obligations with the same pair of participants and opposite positions.*

Proof. Proof see Appendix A. ∎

The efficiency of bilateral compression is straightforward. It corresponds to the effect of netting out each pair of bilateral exposures. We thus obtain the following efficiency results:

Proposition 10. *Given a market $G = (N, \mathbf{E})$, a compression operator $c^b()$ solving the bilateral compression problem eliminates a level of excess equal to $\sum_{i,j \in N} \min\{e_{ij}, e_{ji}\}$ where $e_{ij}, e_{ji} \in \mathbf{E}$.*

Proof. Proof see Appendix A. ∎

Technically, bilateral compression results in the removal of all closed chains of intermediation of length two. Hence, a bilaterally compressed market exhibits a maximum of one obligation between each pair of market participants.

3.3 Compression efficiency ranking

We close this Section with a ranking of efficiencies. For each setting, we consider the maximum amount of excess that can be eliminated.

In order to compare efficiencies under different tolerance sets, we associate each compression operator $c^k(G)$ with its relative level of excess reduction $\rho_k = \frac{\Delta(G) - \Delta(c^k(G))}{\Delta(G)}$. A compression operator over a market G, $c^s(G)$, is therefore more efficient than another compression operator, $c^t(G)$, if $\rho_s > \rho_t$. This efficiency ratio is invariant to scale transformations allowing for treatments such as exchange rate effects (see Appendix F for details and derivations).

Proposition 11. *Given a market $G = (N, \mathbf{E})$ and the set of compression operators $\{c^c(), c^{nc}(), c^h(), c^b()\}$ such that:*

- *ρ_c is the efficiency of $c^c()$ which solves the conservative compression problem,*
- *ρ_{nc} is the efficiency of $c^{nc}()$ which solves the non-conservative compression problem,*
- *ρ_h is the efficiency of $c^h()$ which solves the hybrid compression problem,*

- *ρ_b is the efficiency of $c^b()$ which solves the bilateral compression problem,*

the following weak dominance holds:

$$\rho_b \leq \rho_c \leq \rho_h \leq \rho_{nc} = 1$$

Proof. Proof see Appendix A. ∎

This result shows a precise dominance sequence. First, we see that non-conservative compression is the most efficient. This stems from the fact that a global non-conservative solution always eliminates all the excess in a market (see Proposition 4). The second most efficient compression operator is the hybrid compression, followed by the conservative. The least efficient approach is the bilateral compression. The loss in efficiency is due to the fact that bilateral compression cannot eliminate excess resulting from chains of length higher than two. The proof of this proposition derives from an analysis of the compression tolerance sets of each problem. In fact, it can be shown that the bilateral compression tolerance set is a subset of the conservative set which in turn is a subset of the hybrid set which is also a subset of the non-conservative set. This nested structure of compression tolerances ensures that any globally optimal solution of a superset is at least as efficient as the globally optimal solution of any subset.

Overall, this result shows a trade-off between efficiency in excess elimination and tolerances relative to changes in the underlying the web of outstanding obligations. The sequence from non-conservative compression to bilateral compression is a discrete gradient of relationship preservation. The more (resp. less) conservative, the less (resp. more) efficient.

Further analysis on the relative efficiencies of each approach (e.g., strong dominance, quantities, etc.) needs to include more detailed information on the underlying matrix of obligations **E**. Therefore, we proceed next with an empirical estimation based on transaction-level data.

4 The data

The dataset used in this paper covers all OTC CDS transactions and positions outstanding from October 2014 to April 2016 in which at least one counterparty is legally based in the

European Union.[7] Our goal is to quantify the full extent of outstanding netting opportunities that a system-wide portfolio compression could exploit. We mainly rely on two assumptions which vary in practice today.

The first assumption is the full participation of market participants in portfolio compression. Rates of adoption of portfolio compression remain currently unequal across markets and participants. In particular, for the markets we analyse, only a small subset of participants engaged in portfolio compression at the time of observation: Appendix G reports an average adoption rate of less than 10% of participants for any given market in our sample. Several barriers may be responsible for this low rate. At the extensive margin, some counterparties may choose not to participate due to different incentives structures: the organisational costs (e.g., collecting and reconciling portfolio information, submitting trades and assessing compression solutions) may outweigh the netting benefits, in particular for financial institutions not subject to strong regulatory requirements (see Appendix B). At the intensive margin, counterparties may be selective in the trades they disclose to their service provider as compression implies possible shifting of trading relationships. Further, hedging strategies associated with specific positions may also prevent some forms of netting to be accepted by individual participants (Donaldson and Piacentino, 2018).

A second assumption is the absence of frictions in the organisation of the market for portfolio compression. Portfolio compression entails network effects: a fragmented market composed of competing service providers induces losses of netting opportunities because trades no longer net between clients of different providers. Our exercise maximizes these network effects by implicitly considering a single service provider with complete information on each bilateral position.

These barriers help explain why the following exercise may find further netting opportunities

[7]Credit default swap contracts are the most used types of credit derivatives. A CDS offers protection to the buyer of the contract against the default of an underlying reference. The seller thus assumes a transfer of credit risk from the buyer. The reason we focus on the CDS market is fourfold. First, CDS contracts are a major instrument to transfer risk in the financial system. The key role they played in the unfolding of the GFC dramatically illustrates this point. Second, CDS markets have been early adopters of portfolio compression as discussed in B. Third, the CDS markets we study are not subject to mandatory clearing and clearing rates remain low (see Abad et al. (2016)). As such, they have maintained a dealer-customer structure relevant for non-trivial compression results. By the same token, they also lend themselves adequately to constructing central clearing counterfactuals as presented in Section 6. Fourth, the nature of these swaps makes them the ideal candidate for our analysis. The notional amount of any bilateral contract corresponds to the expected payment (minus recovery rate) from the seller of protection to the buyer in case of default of the underlying entity. Therefore, we assume a reasonable amount of fungibility between positions written on the same reference entity with the same maturity date. In addition, it is always possible to identify, at any point in time, the payer and the receiver. For other types of swaps, such as interest rate swaps, payer and receiver may change during the lifetime of a given trade and the overall analysis becomes less straightforward.

despite partial adoption in some of the markets. Given the networked nature of the problem, lack of (or partial) participation from critical participants in incomplete settings may result in important losses of netting opportunities compared to our benchmarks.

From a policy perspective, the following analysis thereby contributes by informing on the potential impact of a deeper penetration of portfolio compression in OTC markets should some of the barriers be lifted through, for instance, a harmonization of regulatory requirements for OTC trading across types of institutions. Such exercise echoes recent debates among policymakers regarding the necessity to support further adoption of portfolio compression as illustrated by the recent consultation from the European Securities and Market Authority on the matter (ESMA, 2020).

4.1 Dataset description and methods

Under EMIR, any legal entity based in the EU is required to report all derivatives trading activity to a trade repository, effective since 2014 (see Section B). Access to this unique dataset allows us to (i) provide the first empirical account of the levels of market excess and (ii) compute the performance of various compression scenarios. Such scenarios consist of estimating structural changes to OTC markets in presence of a market wide adoption of the compression technology and subject to the different sets of tolerances introduced in Section 3.

We use 19 mid-month snapshots from October 2014 to April 2016. Overall, the original sample comprises 7,300 reference entities. The vast majority of the notional, however, is concentrated in a lower number of reference entities. We retain the top 100 reference entities upon which CDS contracts are written and which we find to be an acceptable compromise between the amount of notional traded and clarity of analysis (see statistics in Section 4.2).

For each reference entity k, a market is the set of outstanding obligations written on k. Each bilateral position reports the identity of the two counterparties, the underlying reference entity, the maturity, the currency and its notional amount. We select the most traded reference identifier associated with the reference with the most traded maturity at each point in time. At the participant level, we select participants using their Legal Entity Identifier (LEI). In practice, financial groups may decide to submit positions coming from different legal entities of the same group. We do not consider such case in the remainder.[8]

Our restricted sample consisting of the 100 most traded reference entities comprises 43 sovereign entities (including the largest EU and G20 sovereign entities), 27 financials (including the largest banking groups) and 30 non-financials entities (including large industrial and manufacturing groups). We analyze each market separately.

For each market, we compute the (i) dealer-customer network characteristics, (ii) excess statistics and (iii) efficiency under each tolerance setting: bilateral, conservative and hybrid compression. We do not report results from non-conservative compression as an optimal solution always leads to zero residual excess by virtue of Proposition 4. In the case of the conservative and hybrid compressions, we design a linear programming framework parametrised to the respective tolerance sets. For each market G, we implement each compression algorithm and compute its efficiency as in Section 3.3.

The resulting efficiency differences allows us to quantify i) the effect of coordinated multilateral compression (i.e., conservative and hybrid cases) versus independent bilateral compression (i.e., bilateral case) and ii) the quantitative effect of relaxing compression tolerances from bilateral to conservative to hybrid settings. In Appendix I, we report the same analysis for bilaterally compressed markets in order to quantify excess and compression efficiency beyond the bilateral redundancy. Results from this Section remain qualitatively robust.

Finally, we compare results from applying multilateral compression on the original market and on the bilaterally compressed market. Doing so quantifies the potential losses in efficiency due to a sequence of bilateral-then-multilateral compression which bears policy design implications.

4.2 Summary statistics

Table 1 provides the main statistics of each market segment. Sampling statistics of the data are reported in the Appendix H. The total notional of the selected 100 markets varies between 380Bn Euros and 480Bn Euros retaining roughly $30 - 34\%$ of the original total gross notional. We compute the average number of dealers, customers on the buy side and customers on the

[8] Our approach is in line with the recent Opinion on Portfolio Margining Requirements under Article 27 of EMIR Delegated Regulation of the European Securities and Market Authority (ESMA). Under articles 28, the netting sets related to different single names and indexes should be separated for portfolio margining comprise. Note that under article 29, different maturities can be considered for the same product which is less conservative than in our approach.

sell side across all entities in the different snapshots. We observe stable numbers across time: per reference entity, there are on average 18 to 19 dealers, 12 to 17 customers buying a CDS, 14 to 21 customers selling a CDS. The average number of bilateral positions per reference entity varies more through time but remains between 140 and 170. Taken as a whole, markets are quite sparse with an average network density around 0.10. This measure is almost three times higher when we only consider the intra-dealer market. The amount of intra-dealer notional also highlights the level of activity concentration around dealers: it averages around 80% of the total notional. These results are in line with the literature (see Section 1). They provide evidence of the tightly-knit structure present in the intra-dealer segment. Finally, the last column of Table 1 confirms the very low frequency of customer-customer trades which, on average, account for less then 0.2% of all obligations.

5 Market excess and compression efficiency

We start by measuring the level of excess present in the original markets as a function of the total gross notional (i.e., $\epsilon(G) = \frac{\Delta(G)}{x}$). Table 2 reports the statistics of excess computed across all reference entities for six snapshots equally spread between October 2014 and April 2016 including minimum, maximum, mean, standard deviation and quartiles, computed across all 100 reference entities in our sample. Results on the means and medians are stable over time and mostly higher than 0.75. The interpretation of this result is that around three quarters of the gross notional in the most traded CDS markets by EU institutions is in excess vis-á-vis participants' net position. At the extremes, we note a high degree of variability: the minimum and maximum levels of excess relative to total gross notional oscillate around 45% and 90% respectively.

Overall, results reported in Table 2 show that large amounts of notional are eligible for compression. We now move to the efficiency of each compression operator. The results are reported in Table 3. After having implemented the compression algorithms on each market, we compute efficiency as defined in Section **??**.

Analyzing the means and medians, we observe that the bilateral compression already removes 50% of excess on average. Nevertheless both multilateral compression approaches (i.e., conservative and hybrid) outperform it by removing around 85% and 90% of the excess re-

Time	Avg. num. dealers	Avg. num. customers buying	Avg. num. customers selling	Avg. num. obligations	Avg. share intra dealer notional	Avg. density	Avg. intra dealer density	Avg. intra customer density
Oct-14	18	16	20	153.72	0.812	0.105	0.332	0.0010
Nov-14	18	16	21	162.26	0.831	0.109	0.345	0.0006
Dec-14	19	17	21	171.13	0.829	0.109	0.339	0.0005
Jan-15	19.	17	21	171.25	0.827	0.106	0.334	0.0006
Feb-15	19	17	21	168.45	0.826	0.106	0.335	0.0004
Mar-15	18	15	17	154.42	0.832	0.110	0.339	0.0007
Apr-15	18	13	15	143.71	0.829	0.110	0.344	0.0005
May-15	18	12	15	143.66	0.827	0.108	0.336	0.0008
Jun-15	18	13	14	142.01	0.828	0.106	0.323	0.001
Jul-15	18	14	14	143.68	0.813	0.101	0.314	0.0009
Aug-15	19	15	17	149.12	0.821	0.101	0.308	0.0011
Sep-15	19	16	17	151.36	0.804	0.098	0.302	0.0018
Oct-15	19	16	18	155.52	0.815	0.099	0.297	0.0013
Nov-15	19	17	19	158.10	0.810	0.099	0.293	0.0017
Dec-15	19	17	19	158.78	0.821	0.098	0.292	0.0012
Jan-16	20	17	18	159.58	0.822	0.098	0.291	0.0013
Feb-16	19	17	18	159.80	0.813	0.098	0.291	0.0012
Mar-16	18	14	17	144.11	0.790	0.096	0.301	0.0018
Apr-16	19	14	17	146.42	0.811	0.098	0.301	0.0019

Table 1: Statistics of sampled markets over time: average numbers of dealers, customers, obligation and concentration statistics.

Total Excess	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.529	0.513	0.475	0.420	0.533	0.403	0.532
max	0.904	0.914	0.895	0.901	0.903	0.890	0.869
mean	0.769	0.777	0.766	0.757	0.751	0.728	0.734
stdev	0.077	0.082	0.085	0.090	0.082	0.096	0.080
first quart.	0.719	0.733	0.712	0.703	0.693	0.660	0.678
median	0.781	0.791	0.783	0.769	0.758	0.741	0.749
third quart.	0.826	0.847	0.832	0.822	0.808	0.802	0.796

Table 2: Statistics of market excess over time: share of notional in excess against total gross notional for each market.

spectively. Levels are larger than the maximum efficiency achievable by bilateral compression which oscillates around 75%. In comparison with the bilateral efficiency, the conservative and hybrid approaches perform similarly on the extremes: minima range between 55% and 62% and maxima range between 98% and 99%, respectively. In particular, results from the conservative compression show that, even under rather stringent constraints, the vast majority of market's excess can be eliminated. This result is made possible by the large levels of concentrations and the tightly-knit structure exhibited in the intra-dealer segment.

From Lemma 2 and Proposition 5, it follows that the more homogeneous and connected a network is, the more conservative and non-conservative performances will converge. In the extreme case of cycles composed of contracts of equal value, conservative compression removes all the excess, thereby performing at par with non-conservative and hybrid compressions. Given our data, conservative compression performs well because OTC markets - as illustrated by Figure 2 - exhibit two enabling features: (i) the intra-dealer segment is well connected and (ii) most of the excess lies in the intra-dealer segment (see Table 1). Furthermore, comparing the relative performances of conservative and hybrid compression algorithms, the same logic suggests that lifting conservative constraints in the intra-dealer segment produces limited marginal gains because intra-dealer netting opportunities are already well exploited under conservative tolerances.

The interplay between bilateral and multilateral compression showcases the added-value of multilateral compression services. In fact, participants can engage in a decentralized and asynchronous fashion to achieve bilateral compression. This is not straightforward for multilateral compression. The difference also allows participants to seek to bilaterally compress some of their positions before participating in a multilateral compression cycle. We analyze this situation as

Bilateral (ρ_b)	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.278	0.281	0.286	0.277	0.276	0.276	0.260
max	0.779	0.791	0.759	0.777	0.717	0.711	0.746
mean	0.528	0.536	0.524	0.522	0.513	0.512	0.543
stdev	0.101	0.106	0.103	0.105	0.107	0.109	0.108
first quart.	0.464	0.460	0.469	0.452	0.448	0.444	0.448
median	0.526	0.542	0.535	0.530	0.517	0.528	0.555
third quart.	0.583	0.597	0.590	0.600	0.596	0.597	0.623

Conservative (ρ_c)	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.558	0.547	0.545	0.507	0.491	0.528	0.574
max	0.985	0.982	0.973	0.967	0.968	0.979	0.969
mean	0.836	0.857	0.848	0.843	0.828	0.827	0.834
stdev	0.091	0.087	0.090	0.091	0.104	0.106	0.090
first quart.	0.781	0.816	0.810	0.800	0.777	0.773	0.788
median	0.852	0.880	0.868	0.858	0.849	0.847	0.860
third quart.	0.906	0.925	0.913	0.915	0.902	0.907	0.904

Hybrid (ρ_h)	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.589	0.626	0.636	0.653	0.574	0.619	0.676
max	0.990	0.994	0.988	0.990	0.994	0.989	0.990
mean	0.878	0.898	0.894	0.893	0.881	0.882	0.898
stdev	0.079	0.072	0.074	0.073	0.085	0.080	0.069
first quart.	0.821	0.859	0.862	0.865	0.831	0.836	0.863
median	0.894	0.916	0.918	0.912	0.901	0.908	0.911
third quart.	0.935	0.952	0.947	0.951	0.948	0.945	0.947

Table 3: Statistics of compression efficiency over time: share excess eliminated after compression against original level of market excess for each market.

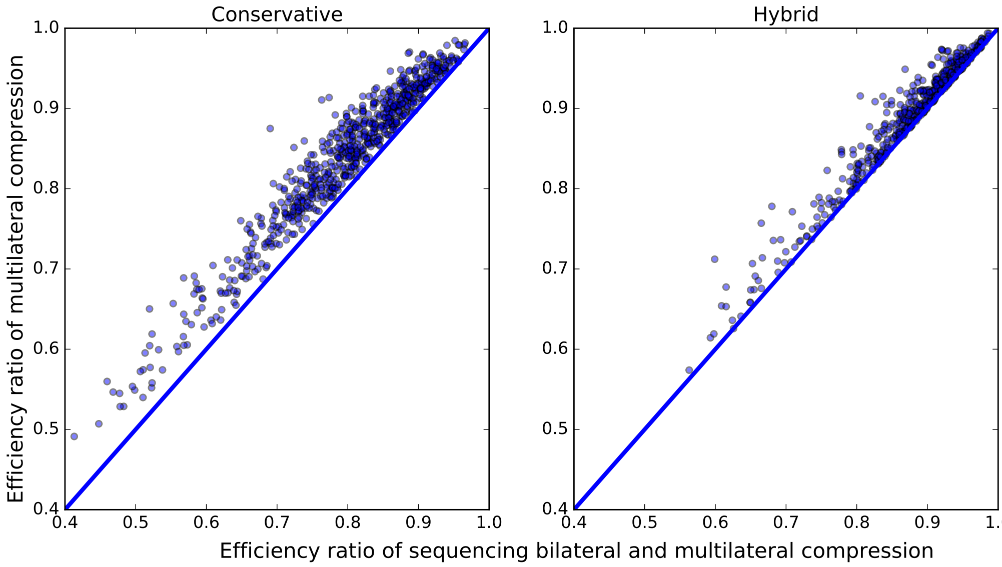


Figure 3: Comparison of the efficiency between multilateral compression in the original markets and a sequence of bilateral and multilateral compression. All snapshots and market instances are reported on the same figures.

follows: for each setting, we compare the efficiency of the operation on the original market with the aggregate efficiency when bilateral compression is applied first.

Figure 3 reports the distribution of efficiency ratios when multilateral compression operators are applied to the full network and when they are combined with bilateral compression first. The latter results are obtained by adding the absolute bilateral results reported in Table 3 to the absolute excess reduction for the conservative and hybrid approach as in Table 10 then dividing by the aggregate notional of the original markets.

The results show that multilateral compression on the original market is always more efficient than the sequence of bilateral-then-multilateral compression. Nevertheless, the sequence is particularly relevant under a conservative setting of preferences. In fact, the difference for hybrid compression is lower (i.e., about one percentage point improvement in the median) than in the conservative case (i.e., up to seven percentage points).

More in general, Figure 3 suggests that a more coordinated and collective action for compression provides more efficiency. Henceforth, regulatory incentives would be more effective when favoring multilateral over bilateral compression. However, under EMIR, while there is no explicit distinction, the condition is set at the bilateral level (i.e., 500 bilateral contracts with the same counterparty), which may encourage bilateral compression. In contrast, measures based on notional approaches such as net-to-gross ratios would potentially improve incentives to compress as well as the efficiency of the multilateral exercises.

6 The effects of clearinghouse proliferation

The promotion of central clearing in OTC derivatives markets has been a major element of the post-crisis regulatory reform (FSB, 2017, 2018). Central clearing consists of interposing a Central Clearing Counterparty (CCP) between each side of a contract. The guiding principle of the reform is based on the premise that increased clearing of transactions provides more stability to markets by means of counterparty risk elimination, increased netting efficiencies and risk mutualization (Cecchetti et al., 2009).

In this section, we adapt our framework to estimate compression performances in the presence of central clearing. In fact, a CCP generates multilateral netting opportunities by pooling cash flows from multiple clearing members. However, when several CCPs are present in a given market, clearing fragmentation entails a social cost due to losses of netting opportunities across members of different CCPs (Benos et al., 2019). In contrast, portfolio compression exploits netting opportunities generated from the original set of bilateral trades. We thus analyse how a re-arrangement of trades through central clearing compares and interacts with the efficiency of portfolio compression, in particular, when the number of CCPs in a given market increases.

Note that, in practice, portfolio compression nets by reducing gross positions while CCPs may rather net by reducing bilateral cash flows with the clearing members. Absent any information or operational friction, netting cash flows and netting gross positions should be equivalent from a systemic risk perspective. For sake of consistency in the comparative statistics that follow, we consider the netting efficiency of central clearing by computing excess reduction when trades are bilaterally compressed with the CCP.

6.1 A single CCP

Introducing a single CCP transforms the network structure of a market into a star network where the CCP, denoted by c, is on one side of all obligations. Every original trade is novated into two new trades. By construction, the CCP has a net position of 0 and its gross position is equal the total market size: $v_c^{gross} = \sum_{c,j} e^{'}_{cj} = x$. Before the bilateral compression with the CCP, we have $x^{CCP} = 2x \quad \text{and} \quad v_i^c = v_i \; \forall i \in N$. In fact, the total size of the market doubles with a CCP while all market participants keep their net position unchanged. Let m be the minimum total notional required to satisfy every participants' net position as defined in Eq 1 from Proposition 1. Hence, the excess before compression is given by: $\Delta(G^{CCP}) = 2x - m = x + \Delta(G)$.

Compression in a single CCP market is equivalent to the bilateral compression of a star-network. All trades between a counterparty i and the CCP c are bilaterally netted such that: $e'_{ic} = max\{e_{ic} - e_{ci}, 0\}$ and $e'_{ci} = max\{e_{ci} - e_{ic}, 0\}$. As a result, the total size of the market after compression with a single CCP is given by $x'(G) = 2m$. Compressing the original market $G = (N, \mathbf{E})$ under one single CCP thus leads to an amount of eliminated excess of $x - x'(G) = x - 2m$. In line with Section 3.3., we can thus compute the efficiency ratio as follows $\rho^{CCP} = \frac{x-2m}{\Delta(G)} = \frac{x-2m}{x-m} = 1 - \frac{m}{x-m}$.

Without loss of generality, we formulate the efficiency under one single CCP as follows:

Proposition 12.

$$\rho^{CCP} = 1 - \frac{m + x - x}{x - m} = 2 - \frac{x}{x - m} = 2 - \frac{1}{\epsilon} \tag{3}$$

where ϵ is the share of excess present in the original market: $\epsilon = \frac{\Delta(G)}{x} = \frac{x-m}{x}$.

From this expression we see that:

Corollary 4.

- *If the excess in a market is less than 50% of total notional, compressing with a single CCP is* ***counter-efficient****: it increases the excess*
- *If the excess in a market is equal to 50% of total notional, compressing with a single CCP is* ***neutral****: it does not modify the excess*
- *If the excess in a market is higher to 50% of total notional, compressing with a single CCP is* ***sub-excess ratio efficient****: the efficiency is always lower than the excess share.*
- *If the excess in a market is equal to 100% of total notional, compressing with a single CCP is* ***fully efficient****: it removes all the excess.*

From the outcomes presented in Corollary 4, we identify the most empirically relevant case using the data described in Section 4. We compare the efficiency of one single CCP with the efficiency results from the previous Section. For each compression setting, we collect the full set of markets - through references and time - and compare the efficiency ratios with Equation 3.

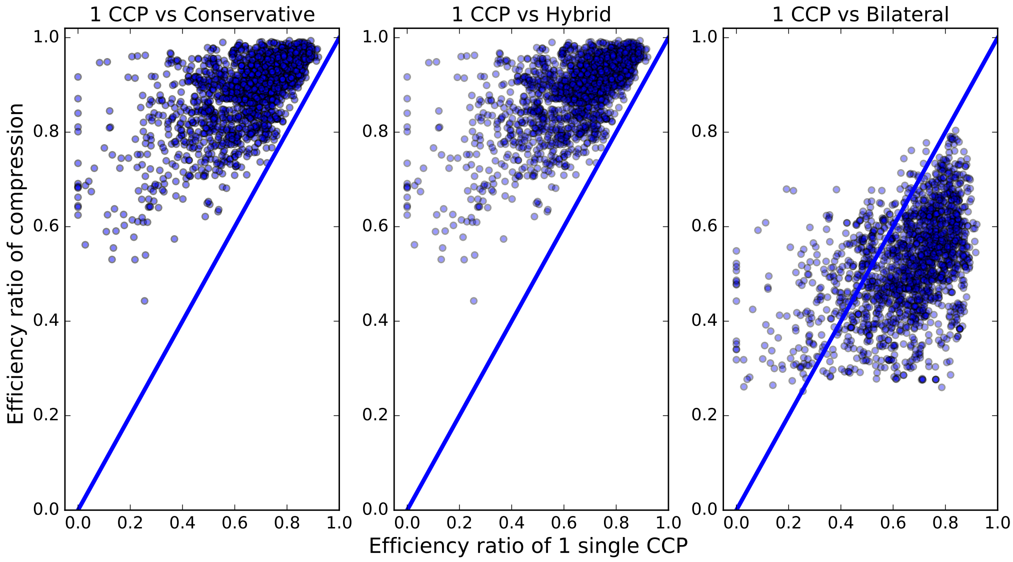


Figure 4: Comparison of efficiency ratios between compression operators and one single CCP. All snapshots and market instances are reported on the same figures.

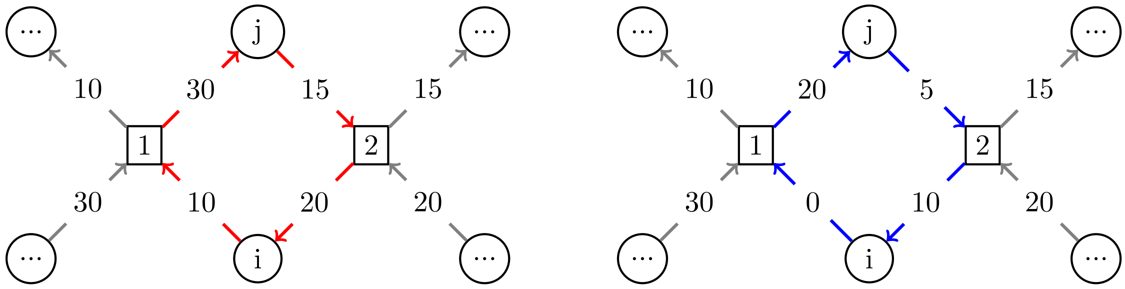
Figure 4 reports the results. The multilateral compression operations (conservative and hybrid) systematically yield higher efficiency than the compression with one single CCP. In the majority of cases, bilateral compression is less efficient than one single CCP.

Despite the multilateral netting opportunities brought by centralization, novating contracts to the clearinghouse also duplicates the notional value of each bilateral obligations. When only considering the effects over gross notional, the above empirical exercise indicates that this trade-off is first-order dominated by multilateral compression without central clearing.

6.2 Multiple CCPs

We consider the case of multiple CCPs in the market. We run an empirical exercise in which the set of bilateral positions is reorganized among several CCPs. For a number n^{ccp} of CCPs, each bilateral position is cleared with one CCP chosen uniformly at random. Once all bilateral positions are assigned and duplicated, each CCP compresses bilaterally with their members. For each given market and n^{ccp}, we generate 1000 realizations of CCP allocations and compute statistics of the compression efficiency ratios.

We study two cases. In the first scenario, only bilateral compression between members and their CCPs can take place. In the second scenario, we analyze the effect of multilateral compression across CCPs. Compression across CCPs can take place when members of one CCP



(a) Before Compression (b) After Compression

Figure 5: A graphical example of portfolio compression across CCPs. Panel (a) exhibits a market consisting of market participants (i, j,and other circles) and CCPs (1 and 2). Panel (b) shows a multilateral compression solution to the market.

are also members of another CCP. In the following, we assume a conservative preference setting among CCPs. Figure 5 provides a stylized example of compression across two CCPs. Note that compression tolerances on counterparty relationships among participants become irrelevant in this context: all participants are exclusively exposed to CCPs.

Table 4 and 5 report the results of the exercise for the five markets with the largest aggregate notional on the last day of our time window for the two scenarios, respectively. First, we observe that an increase in the number of CCPs has vast adverse effects on the elimination of excess as shown in Table 4. The proliferation of CCPs reorganizes the web of obligation creating separated segments around each CCP. Global netting opportunities are dramatically lost at the bilateral level. Whereas the single CCP configuration was sufficiently efficient to compensate for the duplication of aggregate notional, this balance does not hold once the number of CCPs increase.

Second, we find that the adverse effect of proliferation is almost entirely offset when obligations can be compressed multilaterally across CCPs as reported in Table 5. Netting opportunities are recovered once the compression exercise includes several CCPs. In particular, proliferation beyond two CCPs yields levels very close to the single CCP scenario.

Note that we have assumed a uniform distribution of trades among CCPs which entails equivalent market shares. In general, increasing concentration to some CCPs should reduce the adverse effects. Nevertheless, the results on cross CCP compression would still hold qualitatively.

	Market 1	Market 2	Market 3	Market 4	Market 5
Original excess (Bil EUR)	34.834	27.489	31.592	26.227	27.051
Conservative compression efficiency	0.924	0.906	0.954	0.927	0.911
Clearing without multilateral compression					
1 CCP	0.805	0.655	0.834	0.71	0.793
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
2 CCPs	0.66	0.543	0.672	0.579	0.716
	(0.056)	(0.036)	(0.048)	(0.044)	(0.025)
3 CCPs	0.576	0.477	0.569	0.502	0.663
	(0.055)	(0.039)	(0.047)	(0.047)	(0.024)
4 CCPs	0.497	0.407	0.482	0.437	0.616
	(0.062)	(0.042)	(0.055)	(0.049)	(0.028)
5 CCPs	0.435	0.364	0.42	0.374	0.588
	(0.059)	(0.04)	(0.055)	(0.051)	(0.029)
6 CCPs	0.385	0.311	0.354	0.317	0.554
	(0.056)	(0.046)	(0.058)	(0.043)	(0.025)
7 CCPs	0.34	0.271	0.302	0.265	0.526
	(0.057)	(0.048)	(0.055)	(0.05)	(0.03)
8 CCPs	0.287	0.226	0.247	0.229	0.495
	(0.058)	(0.05)	(0.057)	(0.049)	(0.028)
9 CCPs	0.239	0.191	0.214	0.176	0.472
	(0.059)	(0.047)	(0.051)	(0.05)	(0.028)
10 CCPs	0.193	0.157	0.166	0.145	0.447
	(0.055)	(0.048)	(0.056)	(0.05)	(0.028)

Table 4: Effect of bilateral compression between participants and CCPs with an increasing number of CCPs. Columns report the average efficiency ratio and standard deviation in parentheses for the 5 markets with the largest notional amounts outstanding on April 15, 2016.

	Market 1	Market 2	Market 3	Market 4	Market 5
Original excess (Bil EUR)	34.834	27.489	31.592	26.227	27.051
Conservative compression efficiency	0.924	0.906	0.954	0.927	0.911
Clearing with multilateral compression					
1 CCP	0.805	0.655	0.834	0.71	0.793
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
2 CCPs	0.774	0.632	0.807	0.682	0.774
	(0.021)	(0.016)	(0.024)	(0.021)	(0.014)
3 CCPs	0.785	0.633	0.81	0.694	0.779
	(0.022)	(0.019)	(0.022)	(0.016)	(0.012)
4 CCPs	0.794	0.638	0.816	0.695	0.787
	(0.016)	(0.02)	(0.018)	(0.017)	(0.009)
5 CCPs	0.797	0.64	0.82	0.7	0.785
	(0.011)	(0.017)	(0.017)	(0.012)	(0.01)
6 CCPs	0.798	0.644	0.826	0.702	0.787
	(0.012)	(0.014)	(0.012)	(0.013)	0(0.008)
7 CCPs	0.798	0.646	0.825	0.705	0.789
	(0.011)	(0.012)	(0.011)	(0.008)	(0.006)
8 CCPs	0.802	0.648	0.827	0.705	0.788
	(0.006)	(0.011)	(0.011)	(0.008)	(0.007)
9 CCPs	0.802	0.651	0.829	0.705	0.788
	(0.007)	(0.007)	(0.01)	(0.009)	(0.007)
10 CCPs	0.802	0.648	0.829	0.706	0.786
	(0.007)	(0.011)	(0.008)	(0.008)	(0.01)

Table 5: Effect of multilateral compression across CCPs with an increasing number of CCPs. Columns report the average efficiency ratio and standard deviation in parentheses for the 5 markets with the largest notional amounts outstanding on April 15, 2016.

6.3 Discussion

Mandates and increased incentives to clear are at the heart of the regulatory response to the GFC. Central clearing and portfolio compression are both post-trade technologies which have reshaped the organization of OTC markets. However their interplay has been so far unclear. We provide here a simple intuition. CCPs provide natural netting opportunities. While exposures towards CCPs are admittedly of a different nature than OTC exposures, concerns about risk concentration and resilience have been raised (Duffie and Zhu, 2011). In turn, the proliferation of CCPs has brought several concerns including liquidation costs, interoperability, cross-border issues and losses in netting efficiency (Glasserman et al., 2015; Ghamami and Glasserman, 2017). In this respect, we investigated the effect of central clearing on the aggregate notional amounts of OTC markets.

The results of our stylized exercise show that a proliferation of CCPs has adverse effects on netting opportunities. Furthermore, our findings show that multilateral compression across CCPs can almost entirely alleviate this concern. This result supports interoperability policies favoring the adoption of compression by CCPs and their mutual participation to multilateral cycles.

7 Concluding remarks

The post-crisis regulatory reforms have generated demand for new post-trade services such as portfolio compression in financial markets (FSB, 2017). This particular multilateral netting technique, which allows market participants to eliminate direct and indirect offsetting positions, has reportedly been responsible for the large downsizing of major OTC derivatives markets (Aldasoro and Ehlers, 2018).

In this paper, we introduce a framework that empirically supports the large effects attributed to a market wide adoption of portfolio compression. We show that OTC markets with fungible obligations and counterparty risk generate large notional excess: gross volumes can far exceed the level required to satisfy every participants' net position. Dealers acting as intermediaries between customers but also between other dealers are the main determinant for the levels of excess empirically observed in markets.

Using a granular dataset on bilateral obligations resulting from CDS contracts, we find that around 75% of total market sizes is in excess, on average. Furthermore, we find that even when participants are conservative regarding their counterparty relationships, engaging in portfolio compression, on average, eliminates 85% of the excess. Finally, we find that the loss of netting efficiency due to multiple CCPs can be offset when portfolio compression take place across CCPs.

To the best of our knowledge, this work is the first to propose a comprehensive framework to analyze the mechanics of compression in terms of both feasibility and efficiency. In general, the extent to which increasing demand for post-trade services in response to regulatory reforms affects market monitoring, market micro-structure and financial stability constitutes a challenging research agenda. Below, we discuss related implications of our results.

The large amounts of excess observed in markets can be a source of financial instability, in particular in times of crisis (Cecchetti et al. (2009), Acharya and Bisin (2014)). Given the empirical structure of OTC markets, portfolio compression can eliminate most of the excess even under conservative constraints. However, our results show that compression affects both the level and distribution of exposures within the market. While reduction in exposures can mitigate systemic risk, the distributional effects of compression may produce opposite outcomes. In particular, we have shown that conservative compression only affects the portfolios inside the intra-dealer segment, thereby increasing the relative exposure of dealers to customers. Recent work by Veraart (2019) and Schuldenzucker et al. (2018) have adopted the framework introduced in this paper to characterise capital conditions mapping such risk redistribution into an increase in systemic risk. These results are analogous to the adverse effect of partial netting in central clearing shown by Amini et al. (2016). In general, Donaldson and Piacentino (2018) show that maintaining offsetting claims may be rational for banks because of the capacity to dilute transfers. However, the authors show that, when internalised, these individual benefits can generate high levels of interconnectedness which ultimately make the system more fragile in the event of a liquidity crisis. Therefore, once netting strategies are endogenized, the effect of compression appear to become ambiguous. More research in this direction is thus needed to uncover the full picture.

In an OTC derivatives context, an important dimension so far absent from the literature is the major role played by post-crisis mandates on capital and margins. As discussed in B, these regulatory requirements constitute the main driver of compression adoption: by reducing gross exposures, participants are able to free up capital to cover sudden liquidity needs. At the level of the individual participant, such mechanisms could therefore prevent default on other obligations. At the aggregate level, the same mechanisms also reduce inventory costs for dealers thereby increasing the hedging and liquidity capacity of the market as a whole (Duffie, 2018).

Furthermore, the results of our paper also show that the interaction between portfolio compression and central clearing can be detrimental for netting efficiency. In order to assess potential systemic instabilities, it may therefore become crucial to enable netting among multiple clearinghouses to help reduce the impact of large exposures and sudden surge in liquidity needs.

In addition to these dimensions, we believe there are specific elements that should also be accounted for when considering the role of compression for financial stability, including the particular form of compression algorithm and compression tolerances being implemented (e.g., a non-conservative algorithm also introduces changes in counterparty risk by swapping trading relationships); the risk of a coordination failure when the incentives of multiple participants are not aligned; the liquidity distortion brought by observed changes in market size due to compression in place of other economic motives.

In conclusion, the introduction of compression technologies may influence over-the-counter markets in multiple ways. Its impact on systemic risk and market liquidity must therefore be assessed within a cost-benefit analysis over a set of trade-offs whose effects need to be evaluated in conjunction. While a complete analysis falls outside the scope of this paper, our framework can be used to support its implementation in future research.

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College Mathematical Finance seminar, the 2017 WEAI/IBEFA Conference, the MIT Institute for Data, Systems and Society seminar, the SIAM Workshop on Networks, the Second ESRB Annual Conference, the 3rd IWH-FIN-FIRE Workshop, the HEC Montréal Workshop of Measurement and Control of Systemic Risk, the Worcester Polytechnic Institute seminar on Mathematical Finance, the St. Gallen Finance Seminar, the KUL Finance Seminar and the Belgian Financial Research Forum. The empirical part of this work was produced while the authors were visiting the European Systemic Risk Board. We therefore acknowledge the outstanding support from the ESRB Secretariat and the research program on derivatives led by the Joint Expert Group on Interconnectedness of the ESRB, chaired by Laurent Clerc and Alberto Giovannini. The authors declare no relevant or material financial interests that relate to the research described in this paper.

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A Proofs

A.1 Proposition 1

Proof. The proof consists of two steps.

1. First, we show that given a market $G = (N, \mathbf{E})$, we can always find a net-equivalent market G' with total notional of $x'(G)$ as in Equation 1.

   Consider the partition of N into the following disjoint subsets: $N^+ = \{i|v_i^{net} > 0\}, N^- = \{i|v_i^{net} < 0\}$ and $N^0 = \{i|v_i^{net} = 0\}$ (such that $N = N^+ \bigcup N^- \bigcup N^0$). Let $\mathbf{B}$ be a new matrix of obligations of size $N^+ \times N^-$ with elements denoted by b_{ij} such that:

   - The graph is bi-partite: $\forall b_{ij},\ i \in N^+, j \in N^-$;
   - $\sum_j b_{ij} = v_i^{net}, \quad \forall i \in N^+$;
   - $\sum_i b_{ij} = v_j^{net}, \quad \forall j \in N^-$.

   The first condition states that there cannot be edges linking two participants from the same subset: participants of N^+ strictly interact with participants from N^-. The total notional of the market $G' = (N, \mathbf{B})$ is thus given by:

   $$\sum_i \sum_j b_{ij} = \sum_{i \in N^+} v_i^{net} = \sum_{i \in N^-} |v_i^{net}|.$$

   As edges from $\mathbf{B}$ only link two nodes within N (i.e., the system is closed), the sum of all net position is equal to 0: $\sum_i v_i^{net} = 0$. Hence, we have: $\sum_{i \in N^+} v_i^{net} + \sum_{j \in N^-} v_j^{net} = 0$. We see that, in absolute terms, the sum of net positions of each set (N^+ and N^-) are equal: $|\sum_{i \in N^+} v_i^{net}| = |\sum_{j \in N^-} v_j^{net}|$. As all elements in each part have the same sign by construction, we obtain: $\sum_{i \in N^+} |v_i^{net}| = \sum_{j \in N^-} |v_j^{net}|$. As a result, we have: $\sum_{i \in N^+} v_i^{net} = \frac{1}{2}|\sum_{i \in N} v_i^{net}|$.

2. Second, we show that $x'(G)$ is the minimum total notional attainable from a net-equivalent operation over $G = (N, \mathbf{E})$. We proceed by contradiction. Consider $G' = (N, \mathbf{B})$ as defined above and assume there exists a $G^* = (N, \mathbf{B}^*)$ defined as a net-equivalent market to G' such that $\sum_i \sum_j b_{ij}^* < x'(G)$. At the margin, such result can only be obtained by a

reduction of some weight in $\mathbf{B}$: $\exists b^*_{ij} < b_{ij}$. If $\sum_i \sum_j b^*_{ij} < x'(G)$, then there exists at least one node for which this reduction is not compensated and thus $\exists v_i^{*net} < v_i^{net}$. This violates the net-equivalent condition. Hence, $x'(G) = \sum_{i:\ v_i^{net}>0}^{n} v_i^{net}$ is the minimum net equivalent notional.

■

A.2 Lemma 1

Proof. We first prove that if there is at least one dealer in the market, then the excess is strictly positive.

Let us use the following indicator to identify dealers in the market:

$$\delta(i) = \begin{cases} 1 & \text{if } \sum e_{ij}.\sum e_{ji} > 0 \qquad \textbf{(dealer)} \\ 0 & \text{otherwise} \qquad\qquad\qquad\;\; \textbf{(customer)} \end{cases}$$

By definition, $\delta(i) = 1 \Leftrightarrow \sum_j e_{ij} \cdot \sum_j e_{ji} > 0$: a dealer has thus both outgoing and incoming edges. Then it holds that:

$$\delta(i) = 1 \;\; \Rightarrow \;\; v_i^{gross} > |v_i^{net}| \;\; \Leftrightarrow \;\; \sum_j e_{ij} + \sum_j e_{ji} > \left| \sum_j e_{ij} - \sum_j e_{ji} \right| .$$

In contrast, for a customer $\sum_j e_{ij} \cdot \sum_j e_{ji} = 0$ and thus $\delta(i) = 0$. Then it holds that:

$$\delta(i) = 0 \;\; \Rightarrow v_i^{gross} = |v_i^{net}| \;\; \Leftrightarrow \;\; \sum_j e_{ij} + \sum_j e_{ji} = \left| \sum_j e_{ij} - \sum_j e_{ji} \right| .$$

The equality is simply proven by the fact that if i is a customer selling (resp. buying) in the market, then $\sum_j e_{ji} = 0$ (resp. $\sum_j e_{ij} = 0$) and thus both ends of the above equation are equal.

If $G = (N, \mathbf{E})$ has $\sum_{i \in N} \delta(i) = 0$, then all market participants are customers, and we thus have: $v_i^{gross} = |v_i^{net}| \; \forall i \in N$. As a result, the excess is given by

$$\Delta(G) = x - \frac{1}{2} \sum_i \left| v_i^{net} \right| = x - \frac{1}{2} \sum_i \left| v_i^{gross} \right| .$$

As in the proof of Proposition 1, the market we consider is closed (i.e., all edges relate to participants in N) and thus: $\sum_i |v_i^{gross}| = 2x$. We thus have no excess in such market: $\Delta(G) = 0$.

If $G = (N, \mathbf{E})$ has $\sum_{i \in N} \delta(i) > 0$, then some market participants have $v_i^{gross} > |v_i^{net}|$. As a result, the excess is given by:

$$\Delta(G) = x - \frac{1}{2}\sum_i |v_i^{net}| = \frac{1}{2}\sum_i |v_i^{gross}| - \frac{1}{2}\sum_i |v_i^{net}| =$$
$$= \frac{1}{2}\big(\sum_i |v_i^{gross}| - \sum_i |v_i^{net}|\big) > 0$$

We now prove that if the excess in a market is strictly positive, then there must exist at least one dealer.

We prove this by contradiction. Suppose no dealer exists, then a participant i is either a buyer or a seller and $\sum_j e_{ij}. \sum_j eji = 0 \ \forall i$. As such the total out-flow is equal to the total in-flow. By virtue of Proposition 1, this case corresponds to the minimum notional amount that satisfies the net-equivalent condition, implying zero excess. Since this violates the assumption of strictly positive excess, we conclude that there must exist at least one participant such that $\sum_j e_{ij}. \sum_j eji > 0$, i.e. a dealer.

■

A.3 Proposition 2

Proof. For sake of clarity, in the following we only focus the notation on the matrices of obligation for the computation of excess. In general, let us decompose the matrix $\mathbf{E}$ into two $\mathbf{E^1}$ and $\mathbf{E^2}$ such that

$$e_{ij} = e_{ij}^1 + e_{ij}^2 \tag{4}$$

We compute the excess for the matrix e_{ij}:

$$\Delta(N, \mathbf{E}) = \sum_{ij} e_{ij} - 0.5 \sum_i \left| \sum_j (e_{ij} - e_{ji}) \right| . \tag{5}$$

Expanding and substituting 4 into 5, we obtain:

$$
\begin{aligned}
\Delta(N,\mathbf{E}) &= \sum_{ij}(e^1_{ij}+e^2_{ij})+ \\
&\quad -0.5\sum_i\left|\sum_j(e^1_{ij}-e^1_{ji}+e^2_{ji}-e^2_{ji})\right| \\
&= \sum_{ij}e^1_{ij}+\sum_{ij}e^2_{ij}+ \\
&\quad -0.5\sum_i\left|\sum_j(e^1_{ij}-e^1_{ji})+\sum_j(e^2_{ji}-e^2_{ji})\right| \qquad (6)
\end{aligned}
$$

By Jensen's inequality, we have that:

$$
\left|\sum_j(e^1_{ij}-e^1_{ji})+\sum_j(e^2_{ji}-e^2_{ji})\right| \leq \left|\sum_j(e^1_{ij}-e^1_{ji})\right| + \left|\sum_j(e^2_{ji}-e^2_{ji})\right|
$$

therefore from 6 it follows that:

$$
\begin{aligned}
\sum_{ij}e_{ij}-0.5\sum_i\left|\sum_j(e_{ij}-e_{ji})\right| \geq \sum_{ij}e^1_{ij}-0.5\sum_i\left|\sum_j e^1_{ij}-e^1_{ji}\right|+ \\
+\sum_{ij}e^2_{ij}-0.5\sum_i\left|\sum_j e^2_{ij}-e^2_{ji}\right|
\end{aligned}
$$

which proves the claim.

We now identify specific cases under our framework in which the equality relationship holds. Let us decompose the original additivity expression:

$$
\begin{gathered}
\Delta(\mathbf{E}) = \Delta(\mathbf{E^D}) + \Delta(\mathbf{E^C}) \\
\underbrace{\sum_i|\sum_j(e_{ij}-e_{ji})|}_{I} = \underbrace{\sum_i|\sum_j(e^D_{ij}-e^D_{ji})|+\sum_i|\sum_j(e^C_{ij}-e^C_{ji})|}_{II}
\end{gathered}
$$

We can decompose each part in the context of a dealer-customer network. Note that customer-to-customer interactions do not affect excess measurement so they can be omitted without loss of generality. As a result, part II can be expressed as:

$$II = \sum_{d\in D} |\sum_{h\in D}(e^D_{dh} - e^D_{hd})| + \sum_{d\in D} |\sum_{c\in C}(e^C_{dc} - e^C_{cd})| \tag{7}$$

$$= \sum_{d\in D} |\sum_{h\in D}(e^D_{dh} - e^D_{hd})| + \sum_{d\in D} |\sum_{c^+\in C^+} e_{c^+d} - \sum_{c^-\in C^-} e_{dc^-}| \tag{8}$$

where D is the set of dealers, C is the set of all customers and the set C^+ (resp. C^-) includes all customers with positive (resp. negative) net positions: $\{i|v_i^{net} > 0 \text{ and } i \in C\}$ (resp. $\{i|v_i^{net} < 0 \text{ and } i \in C\}$).

Part I can be decomposed as follows:

$$I = \sum_{d\in D} |\sum_{h\in D}(e_{dh} - e_{hd}) + \sum_{c^+\in C^+}(e_{c^+d} - e_{dc^+}) + \sum_{c^-\in C^-}(e_{c^-d} - e_{dc^-})|$$

$$= \sum_{d\in D} |\sum_{h\in D}(e_{dh} - e_{hd}) + \sum_{c^+\in C^+} e_{c^+d} - \sum_{c^-\in C^-} e_{dc^-}|$$

Combining the decomposition of I and II and removing the subscripts related to the different networks, and we obtain the general condition for additive excess:

$$\sum_{d\in D} |\sum_{h\in D}(e_{dh} - e_{hd}) + \sum_{c^+\in C^+} e_{c^+d} - \sum_{c^-\in C^-} e_{dc^-}| = \sum_{d\in D} |\sum_{h\in D}(e_{dh} - e_{hd})| + \sum_{d\in D} |\sum_{c^+\in C^+} e_{c^+d} - \sum_{c^-\in C^-} e_{dc^-}|$$

We can therefore observe that above relationship holds when

1. $\sum_{h\in D}(e_{dh} - e_{hd}) = 0, \quad \forall d \in D$

   or

2. $\sum_{c^+\in C^+} e_{c^+d} - \sum_{c^-\in C^-} e_{dc^-} = 0, \quad \forall d \in D$

■

A.4 Proposition 3

Proof. Non-conservative compression tolerances allow all possible re-arrangements of edges. Hence, the only condition for non-conservative compression to remove excess (i.e., $\Delta_{\text{red}}(c^n(G)) >$

0) is merely that excess is non-zero (i.e., $\Delta(G) > 0$). From Lemma 1, we know that positive excess exists in $G = (N, \mathbf{E})$ only when there is intermediation (i.e., $\exists i \in N | \delta(i) = 1$). ■

A.5 Proposition 4

Proof. We proceed by defining a procedure that satisfies non-conservative compression constraints and show that this procedure (algorithm) generates a new configuration of edges such that the resulting excess is 0.

Similar to the proof of Proposition 1, consider the three disjoint subsets $N^+ = \{i | v_i^{net} > 0\}, N^- = \{i | v_i^{net} < 0\}$ and $N^0 = \{i | v_i^{net} = 0\}$, such that $N = N^+ \bigcup N^- \bigcup N^0$. Let $\mathbf{B}$ be a new matrix of obligations of size $N^+ \times N^-$ with elements denoted by b_{ij} such that:

- The graph is bi-partite: $\forall b_{ij}, \quad i \in N^+, j \in N^-$
- $\sum_j b_{ij} = v_i^{net}, \quad \forall i \in N^+$
- $\sum_i b_{ij} = v_j^{net}, \quad \forall j \in N^-$

The first condition states that there cannot be edges linking two participants from the same subset: participants of N^+ strictly interact with participants from N^-. The market $G' = (N, \mathbf{B})$ is net-equivalent to G while the total gross notional is minimal in virtue of Proposition 1. The nature of the new matrix makes G' bipartite (i.e., $\forall b_{ij}, \ i \in N^+, j \in N^-$), hence, there is no intermediation in G'. The procedure depicted above to obtain $\mathbf{B}$ is a meta-algorithm as it does not define all the steps in order to generate $\mathbf{B}$. As a result, several non-conservative compression operation $c^{nc}()$ can satisfy this procedure consistuting a set C^{nc}. Therefore, by virtue of Proposition 3, for any $c^{nc}() \in C^{nc}$ we have that $\Delta(c^n(G)) = \Delta(G') = 0$. ■

A.6 Proposition 5

Proof. In a conservative compression, we have the constraint:

$$0 \leq e'_{ij} \leq e_{ij}$$

At the individual level, assume i is a customer selling in the market (i.e., $\delta(i) = 0$). Under a conservative approach, it is not possible to compress any of the edges of i. In fact, in order to keep the net position of i constant, any reduction of ε in an edge of i (i.e., $e'_{ij} = e_{ij} - \varepsilon$) requires a change in some other edge (i.e., $e'_{ik} = e_{ik} + \varepsilon$) in order to keep $v_i'^{net} = v_i^{net}$. Such procedure violates the conservative compression tolerance: $e'_{ik} = e_{ik} + \varepsilon > e_{ik}$. The same situation occurs for customers buying. Conservative compression can thus not be applied to node i if $\delta(i) = 0$.

The only configuration in which a reduction of an edge e_{ij} does not require a violation of the conservative approach and the net-equivalence condition is when i can reduce several edges in order to keep its net balance. In fact, for a node i, the net position is constant after a change $\sum_j e'_{ij} = \sum_j e_{ij} - \varepsilon$ if it is compensated by a change $\sum_j e'_{ji} = \sum_j e_{ji} - \varepsilon$. Only dealers can apply such procedure. Furthermore, such procedure can only be applied to links with other dealers: a reduction on one link triggers a cascade of balance adjusting that can only occur if other dealers are concerned as customers are not able to re-balance their net position as shown above. Hence, the amount of eliminated excess for a conservative approach results from intra-dealer links.

Finally, the sequence of rebalancing and link reduction can only stop once it reaches the initiating node back. Hence, conservative compression can only be applied to directed closed chains of intermediation. ■

A.7 Proposition 6

Proof. From Proposition 5, we know that the conservative compression approach can only reduce excess in closed chains of intermediation. Given a market $G = (N, \mathbf{E})$, let $i \in N$ satisfy the following condition:

$$\begin{cases} \sum_j e^C_{ij} > 0 \\ \sum_j e^C_{ji} > 0 \end{cases}$$

The participant i is thus a dealer in the market. More precisely, irrespective of her activity with other dealers (i.e., intra-dealer market $\mathbf{E}^{\mathbf{D}}$), i interacts with customers both on the buy and on the sell side. By definitions, those sets of counterparties generate no excess as they are only active on one side.

As a result, i belongs to open chains of intermediation where customers selling are on the sender's end of the chain while customers buying are on the receiver's end of the chain. By virtue of the conservative setting, it is not possible to compress those open chain as both extreme-ends of the chains are not intermediaries. In turn, the excess generated by those chains cannot be compressed: $\Delta_{\text{res}}(c^c(G)) > 0$.

Assume, instead, that all dealers only interact with one type of customer:

$$\begin{cases} \sum_j e_{ij}^C \cdot \sum_j e_{ji}^C = 0, \\ \sum_j e_{ik}^D \cdot \sum_j e_{ki}^D \geq 0 \end{cases} \qquad \forall i \in N^D$$

In such case, there always exists a configuration of the intra-dealer market such that all the excess can be removed via conservative compression. In fact, if the intra-dealer market is composed of equally weighted closed chains of intermediation, they can all be conservatively compressed out of the market. As a result, only dealer-customer trades with remain. Given the original configuration, no dealer would be intermediating anymore and no excess would be left in the market after consevative compression.

We thus see that in order to ensure positive residual excess after conservative compression, we need open chains of intermediation in the original market which are ensured by the existence of direct intermediation between customers. ■

A.8 Lemma 2

Proof. A conservative compression on a closed chain of intermediation $G^K = (N, \mathbf{K})$ implies that any reduction by an arbitrary $\varepsilon > 0$ on an edge $e'_{ik} = e_{ik} - \varepsilon$ must be applied on all other edges in the chain in order for the compression to be net equivalent (i.e., $v_i^{\prime net} = v_i^{net} \;\; \forall i \in N$): $e'_{ij} = e_{ij} - \varepsilon \;\; \forall e'_{ij} \in \mathbf{K}'$ where $\mathbf{K}'$ is the resulting set of edges: $c^c(G^K) = (N, \mathbf{K}')$.

Overall, reducing by ε one edge therefore leads to an aggregate reduction of $|K|\varepsilon$ after the re-balancing of net positions, where $|K|$ is the total number of edges in $\mathbf{K}$. Recall that, in a conservative compression, we have $0 \leq e'_{ij} \leq e_{ij}$. Hence, for each edge, the maximum value that ε can take is e_{ij}. At the chain level, this constraint is satisfied if and only if $\varepsilon = \min_{e_{ij}}\{\mathbf{K}\}$, that is, ε equals the minimum notional value of edges in $\mathbf{K}$. The total eliminated excess is given by $|K|\varepsilon$. The residual excess is thus given by

$$\Delta(c^c(G^K)) = \Delta(G^K) - |K|\varepsilon \quad \forall c^c \in C$$

■

A.9 Proposition 8

Proof. If $\Delta(N, \mathbf{E}) = \Delta(N, \mathbf{E^D}) + \Delta(N, \mathbf{E^C})$, then we can separate the compression of each market.

Intra-dealer market $(N, \mathbf{E^D})$. According to the hybrid compression, the set of constraints in the intra-dealer market is given by a non-conservative compression tolerances set. According to Proposition 4, the residual excess is zero. We thus have:

$$\Delta(c^h(N, \mathbf{E^D})) = 0.$$

Customer market $(N, \mathbf{E^C})$. According to the hybrid compression, the set of constraints in the customer market is given by a conservative compression tolerances set. Since, by construction, the customer market does not have closed chains of intermediation, it is not possible to reduce the excess on the customer market via conservative compression. We thus have:

$$\Delta(c^h(N, \mathbf{E^C})) = \Delta(N, \mathbf{E^C}).$$

Finally, we obtain

$$\begin{aligned}\Delta(c^h(N, \mathbf{E})) &= \Delta(c^h(N, \mathbf{E^D})) + \Delta(c^h(N, \mathbf{E^C})) \\ &= \Delta(N, \mathbf{E^C})\end{aligned}$$

■

A.10 Proposition 9

Proof. If the market $G = (N, \mathbf{E})$ is such that $\nexists i, j \in N$ *s.t.* $e_{ij}.e_{ji} > 0$ then the compression tolerances will always be:

$$a_{ij} = b_{ij} = \max\{e_{ij} - e_{ji}, 0\} = e_{ij}$$

Hence, $\Delta(G) - \Delta(c^b(G)) = \Delta(G)$ and thus $\Delta(c^b(G)) = 0$. If the market $G = (N, \mathbf{E})$ is such that $\exists i, j \in N$ *s.t.* $e_{ij}.e_{ji} > 0$ then the bilateral compression will yield a market $G' = (N, \mathbf{E}')$ where $x^{'} < x$. Hence, $\Delta(G) - \Delta(c^b(G)) < \Delta(G)$ and thus $\Delta(c^b(G)) > 0$. ■

A.11 Proposition 10

Proof. If the market $G = (N, \mathbf{E})$ is such that $\exists i, j \in N \ s.t. \ e_{ij}.e_{ji} > 0$, then, bilaterally compressing the pair i and j yields the following situation. Before compression, the gross amount on the bilateral pair was $e_{ij} + e_{ji}$. After compression, the gross amount on the same bilateral pair is $|e_{ij} - e_{ji}|$. Hence, we have a reduction of gross notional of $2. \min\{e_{ij}, e_{ji}\}$. The market gross notional after compression of this bilateral pair is thus given by: $x^{'} = x - 2. \min\{e_{ij}, e_{ji}\}$ and the excess in the new market (i.e., residual excess after having bilaterally compressed the pair (i, j)) follows the same change: $\Delta(G) = \Delta(G) - 2. \min\{e_{ij}, e_{ji}\}$. We generalize the result by looping over all pairs and noting that the reduction $\min\{e_{ij}, e_{ji}\}$ is doubled counted: pairing by (i, j) and (j, i). Hence, we reach the following expression of the residual excess:

$$\Delta(c^b(G)) = \Delta(G) - \sum_{i,j \in N} \min\{e_{ij}, e_{ji}\}$$

■

A.12 Proposition 11

Proof. We proceed by analyzing sequential pairs of compression operators and show the pairing dominance before generalizing. We start with the bilateral compressor $c^b()$ and the conservative compressor $c^c()$. Let $(a^b_{ij}, b^b_{ij}) \in \Gamma^b$ and $(a^c_{ij}, b^c_{ij}) \in \Gamma^c$ be the set of compression tolerance for the bilateral and conservative compressor, respectively. We have the following relationship:

$$a^c_{ij} \leq a^b_{ij} = b^b_{ij} \leq b^c_{ij}$$

In fact, by definition of each compression tolerance set, we have:

$$0 \leq \max\{e_{ij} - e_{ji}, 0\} \leq e_{ij}$$

Hence, we see that the set of possible values couple for bilateral compression is bounded below and above by the set of conservative compression values. By virtue of linear composition, a solution of the bilateral compression thus satisfies the conservative compression tolerance set. The other way is not true as the lower bound in the bilateral case a^b_{ij} can be equal to $e_{ij} - e_{ji}$ while in the conservative case, we always have that $a^c_{ij} = 0$. Hence, in terms of efficiency, we have that a globally optimal conservative solution is always at least equal, in amount of eliminated excess, to the globally optimal bilateral solution: $\Delta(c^b(G)) \geq \Delta(c^c(G))$. The case in which the efficiency of $\Delta(c^c(G))$ is higher is a function of the network structure of G. In fact, if the market G only exhibits cycles of length one, we have $\Delta(c^b(G)) = \Delta(c^c(G))$. Once G exhibits higher length cycles, we have a strict dominance $\Delta(c^b(G)) > \Delta(c^c(G))$. Similar reasoning is thus applied to the next pairing: conservative and hybrid compression tolerance sets. Let $(a^h_{ij}, b^h_{ij}) \in \Gamma^h$ be the set of compression tolerance for the hybrid compressor. We have the following nested assembly:

$$a^c_{ij} = a^h_{ij} \quad \text{and} \quad b^c_{ij} = b^h_{ij} \quad \forall e^C_{ij} > 0$$

$$a^c_{ij} = a^h_{ij} \quad \text{and} \quad b^c_{ij} \leq b^h_{ij} \quad \forall e^D_{ij} > 0$$

Where $\mathbf{E^C}$ and $\mathbf{E^D}$ are the customer market and the intra-dealer market, respectively, with $\mathbf{E^C} + \mathbf{E^D} = \mathbf{E}$. In fact, by definition of the compression tolerance sets in the customer market $\mathbf{E^C}$ are the same while for the intra-dealer market we have:

$$a^c_{ij} = a^h_{ij} = 0 \quad \text{and} \quad e_{ij} \leq +\infty \quad \forall e^D_{ij} > 0$$

Similar to the dominance between bilateral and conservative compression, we can thus conclude that: $\Delta(c^c(G)) \geq \Delta(c^h(G))$. It is the relaxation of tolerances in the intra-dealer market that allows the hybrid compression to be more efficient than the conservative compression. By virtue of complementarity of this result, the hybrid and non-conservative pairing is straightforward: $\Delta(c^h(G)) \geq \Delta(c^n(G))$. As we know from Proposition 4, $\Delta(c^n(G)) = 0$, we thus obtain the general formulation of weak dominance between the 4 compression operators:

$$\Delta(c^b(G)) \geq \Delta(c^c(G)) \geq \Delta(c^h(G)) \geq \Delta(c^n(G)) = 0$$

which in turn give:

$$\rho_b \leq \rho_c \leq \rho_h \leq \rho_n = 1$$

■

B Institutional background

In contrast to centrally organized markets where quotes are available to all market participants and exchange rules are explicit, participants in OTC markets trade bilaterally and have to engage in search and bargaining processes. The decentralized nature of these markets makes them opaque as market information is often limited for most agents (Duffie, 2012). These markets were central to the Global Financial Crisis. According to Cecchetti et al. (2009): "Before the crisis, market participants and regulators focused on net risk exposures, which were judged to be comparatively modest. In contrast, less attention was given to the large size of their gross exposures. But the crisis has cast doubt on the apparent safety of firms that have small net exposures associated with large gross positions. As major market-makers suffered severe credit losses, their access to funding declined much faster than nearly anyone expected. As a result, it became increasingly difficult for them to fund market-making activities in OTC derivatives markets – and when that happened, it was the gross exposures that mattered."

As a result, the size, complexity and opacity of OTC derivatives markets have been a key target of the major regulatory reforms following the after-crisis meeting of the G-20 in September 2009. The summit resulted in a commitment to "*make sure our regulatory system for banks and other financial firms reins in the excesses that led to the crisis*" (Art. 16 of the Leader's Statement of the Pittsburgh Summit). This initiative prompted two major financial regulatory reforms: the Dodd-Frank act in the US and the European Market Infrastructure Regulation (EMIR) in Europe. Such reforms include mandatory clearing of specific asset classes and standardized trading activity reports. In addition, the completion of the Basel III accords led to a general increases in capital and collateral requirements, especially regarding uncleared over-the-counter transactions.Formally, the Markets in Financial Instrument Regulation (MiFIR) defines portfolio compression as follows: "Portfolio compression is a risk reduction service in which two or more counterparties wholly or partially terminate some or all of the derivatives submitted by those counterparties for inclusion in the portfolio compression and replace the terminated derivatives with another derivatives whose combined notional value is less than the combined notional value of the terminated derivatives" (see MiFIR, EU Regulation No 600/2014, Article 2 (47)). A similar definition is provided under the Dodd Franck act (see CFTC Regulation 23.500(h)). This set of policy changes generated a large demand for novel services to accommodate the renewed regulatory environment (FSB, 2017). In particular, efficient post-trade portfolio management became crucial to large financial institutions (Duffie, 2017).

Portfolio compression is a post-trade mechanism which exploits multilateral netting opportunities to reduce counterparty risk (i.e., gross exposures) while maintaining similar market risk (i.e., net exposures). The netting of financial agreements is a general process that can encompass different mechanisms. For example, *close-out* netting is a bilateral operation that takes place after the default of one counterparty in order to settle payments on the net flow of obligations. In this respect, portfolio compression can be formally defined as a *multilateral novation netting technique* that does not require the participation of a central clearinghouse. Rather than rejecting the participation of a central clearinghouse, this definition states that compression can be achieved even in the absence of a central counterparty. This distinction is relevant as *multilateral netting* has often been equated only with central clearing. For sake of clarity and consistency with the current industry practices, we choose to articulate to remainder of the paper using the wordings related to *compression*.

Over the last decade, the adoption of portfolio compression in derivatives markets has reportedly brought major changes. According to ISDA (2015) - the International Swaps and Derivatives Association report - portfolio compression is responsible for a reduction of 67% in total gross notional of Interest Rate Swaps. Aldasoro and Ehlers (2018) attributes the reduction of Credit Default Swap notional to a sixth of the levels exhibited a decade before to an extensive use of portfolio compression after the crisis. TriOptima, a leader in the compression business, reports over one quadrillion USD in notional elimination through their services.[9]

The mechanism of portfolio compression can also be seen as a multilateral deleveraging process operated without capital injection nor forced asset sales. Under the capital and collateral requirements resulting from the regulatory reforms, market participants engaging in portfolio compression are able to alleviate capital and collateral needs while preserving their capital structure and net market balances. For instance, capital requirements under the Basel framework are computed including gross derivatives exposures (BIS, 2016). Overall, we observe that the growing adoption of compression services has been driven by both incentives to improve risk management and adapt to the new regulatory requirements.

In practice, multilateral netting opportunities can be identified only once portfolio informa-

[9]Continuous updates are reported in `http://www.trioptima.com/services/triReduce.html`. Last check June 2017.

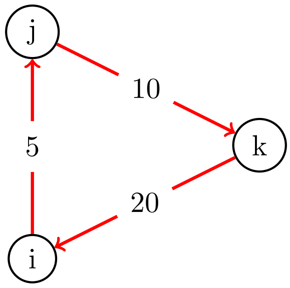


Figure 6: Original configuration the market

tion is obtained from several participants. However, it is individually undesirable for competing financial institutions to disclose such information among each other. Third-party service providers typically come at play to maintain privacy and provide guidance to optimize the outcome. To run a full compression cycle, compression services (i) collect data provided by their clients, (ii) reconstruct the web of obligations amongst them, (iii) identify optimal compression solutions and (iv) generate individual portfolio modification instructions to each client independently.

Portfolio compression has, in general, received a global regulatory support. For example, under the European Market Infrastructure Regulation (EMIR), institutions that trade more than 500 contracts with each other are required to seek to compress their trades at least twice a year. See the Article 14 of Commission Delegated Regulation (EU) No 149/2013 of 19 December 2012 supplementing Regulation (EU) No 648/2012 of the European Parliament and of the Council with regard to regulatory technical standards on indirect clearing arrangements, the clearing obligation, the public register, access to a trading venue, non-financial counterparties, and risk mitigation techniques for OTC derivatives contracts not cleared by a CCP (OJ L 52, 23.2.2013, p. 11- "Commission Delegated Regulation on Clearing Thresholds" or "RTS"). However, research on portfolio compression has been limited. In-depth analyses on the impact of portfolio compression for both markets micro-structure and financial stability has been lacking.

C A simple example with 3 market participants

To better articulate the different ways in which portfolio compression can take place according to the conservative and non-conservative approach, let us take the following example of a market made of 3 participants. Figure 6 graphically reports the financial network: i has an outstanding obligation towards j of notional value 5 while having one from k of notional value 20 and j has an outstanding obligation towards k of notional value 10. For each participant, we compute the gross and net positions:

$$v_i^{gross} = 25 \qquad v_i^{net} = -15$$
$$v_j^{gross} = 15 \qquad v_j^{net} = +5$$
$$v_k^{gross} = 30 \qquad v_k^{net} = +10$$

We also obtain the current excess in the market:

$$\Delta(G) = 35 - 15 = 20$$

Let us first adopt a conservative approach. In this case, we can only reduce or remove currently existing bilateral positions. A solution is to remove the obligation between i and j and adjust the two other obligation accordingly (i.e., subtract the value of ij from the two other obligations). The resulting market is represented in Figure 7(a). Computing the same measurements as before, we obtain:

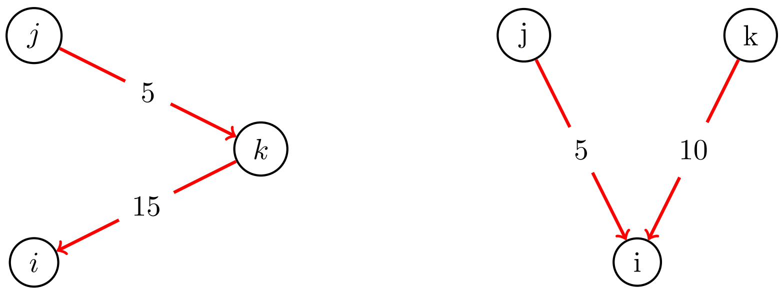
$$v_i^{\prime gross} = 15 \qquad v_i^{\prime net} = -15$$
$$v_j^{\prime gross} = 5 \qquad v_j^{\prime net} = +5$$
$$v_k^{\prime gross} = 20 \qquad v_k^{\prime net} = +10$$

We also obtain the new excess in the market:

$$\Delta^{cons}(G') = 20 - 15 = 5$$

We see that, after applying the conservative compression operator that removed the (i, j) obligation, we have reduced the excess by 15. It is not possible to reduce the total excess further without violating the conservative compression tolerances. We thus conclude that, for the conservative approach, the residual excess is 5 and the amount of eliminated excess is 15.

Let us now go back to the initial situation of Figure 6 and adopt a non-conservative approach. We can now create, if needed, new obligations. A non-conservative solution is to remove trades



(a) After conservative compression (b) After non-conservative compression

Figure 7: Examples of conservative and non-conservative compression approaches.

and create 2: one going from j to i of value 5 and one going from k to i of value 10. We have created an obligation that did not exist before between j and i. The resulting market is depicted in Figure 7(b). Computing the same measurements as before, we obtain:

$$v_i'^{gross} = 15 \qquad v_i'^{net} = -15$$

$$v_j'^{gross} = 5 \qquad v_j'^{net} = +5$$

$$v_k'^{gross} = 10 \qquad v_k'^{net} = +10$$

We also obtain the current excess in the market:

$$\Delta(G') = 15 - 15 = 0$$

We observe that we have eliminated all excess in the resulting market while all the net positions have remained constant. Individual gross positions are now equal to the net positions. Nevertheless the solution has generated a new position (i.e., from j to i). We thus conclude that, for the non-conservative approach, the residual excess is 0 and the amount of eliminated excess is 20.

The results are summarized in Table 6. Though simple, the above exercise hints at several intuitive mechanisms and results that are developed further in the paper.

	Conservative	Non-conservative
Total excess	20	20
Eliminated excess	15	20
Residual excess	5	0

Table 6: Table summarizing the results applying conservative and non-conservative compression on the market with 3 participants in Figure 7.

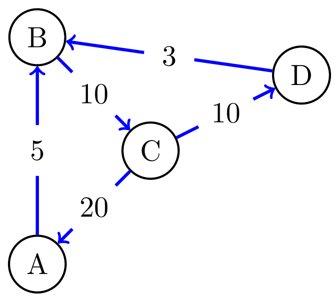


Figure 8: Example of market with entangled chains

D Further analysis on conservative compression

In order to reach a directed acyclic graph any algorithm would need to identify and break all closed chains of intermediation. Nevertheless, the sequences of chains to be compressed can affect the results. In fact, if two chains share edges, compressing one chain modifies the value of obligations also present in the other one. There can be different values of residual excess depending on which closed chain is compressed first.

Formally, we identify such case as a case of *entangled chains of intermediation.*

Definition (Entangled Chains)**.** *Two chains of intermediation,* $\mathbf{K^1}$ *and* $\mathbf{K^2}$*, are entangled if at least for one obligation pair* (i, j) *we have:*

$$e^1_{ij}.e^2_{ij} > 0$$

An illustration of entangled chains is provided in Figure 8 where the edge BC is share by two chains of intermediation (i.e., ABC and BCD).

As such, we formulate the following feature of a graph:

Definition. *(Chain Ordering Proof). A market is chain ordering proof w.r.t. to the conservative compression problem if the ordering of entangled chains does not affect the efficiency of compression.*

If the configuration of entangled chains is such that, according to the initial ordering of excess reduction resulting from a compression on each chain, the optimal sequence is not affected by the effects of compression on other entangled chains, the market is said to be chain ordering proof. Under the above Definition, the optimal conservative compression yields a Directed Acyclic Graph (DAG) where the excess is given by the following expression:

Proposition 13. *Given a market $G = (N, \mathbf{E})$ and a compression operator c^c solving the conservative compression problem, if there are no entangled chains, we have:*

$$\Delta(c^c(G)) = \Delta(G) - \sum_{K_i \in \Pi} |K_i| \varepsilon_i$$

In the presence of entangled chains, (i.e., $G = (N, E)$ is chain-ordering proof), we have

$$\Delta(c^c(G)) > \Delta(G) - \sum_{K_i \in \Pi} |K_i| \varepsilon_i$$

where $\Pi = \{\mathbf{K}_i\}$ is the set of all directed closed chains of intermediation $\mathbf{K}_i$ in $\mathbf{E}$, $\varepsilon_i = \min_e\{\mathbf{K}_i\}$ and $|K_i|$ is the total number of edges in the set $\mathbf{K}_i$

Proof. Proof:

If there are no entangled chains in $G = (N, \mathbf{E})$, then the following conservative procedure:

1. list all closed chains of intermediation $\mathbf{K_i} \in \Pi$ and
2. maximally compress each chain separately,

reaches maximal efficiency. The residual excess is given after aggregating the excess removed on each closed chain separately as given by Lemma 2.

If there are entangled chains but the market $G = (N, \mathbf{E})$ is chain ordering proof, compressing chains separately only provides the upper bound as there will be cases where entangled chains will need to be updated (due to the reduction of one or more edges). Hence, we have $\Delta(c^c(G)) > \Delta(G) - \sum_{K_i \in \Pi} |K_i| \varepsilon_i$. ■

For illustrative purpose, we present an algorithm that always reaches a global solution under the chain ordering proof assumption in the Appendix E.

E Compression algorithms

E.1 Non-Conservative algorithm

In order to provide a rigorous benchmark, we propose a deterministic non-conservative compression algorithm that eliminates all excess.

Data: Original Market $G = (N, \mathbf{E})$

Result: G^* such that $\Delta(G^*) = 0$

Let $N^+ = \{s \text{ s.t. } v_n^s > 0 \text{ and } s \in N\}$ be ordered such that $v_1^{net} > v_2^{net}$;

Let $N^- = \{s \text{ s.t. } v_s^{net} < 0 \text{ and } s \in N\}$ be ordered such that $v_1^{net} > v_2^{net}$;

Let i = 1 and j = 1;

while $i! = |N^+|$ *and* $j! = |N^-|$ **do**

  Create edge $e^*_{ij} = min(v_i^{net} - \sum_{j'<j} e^*_{ij'}, v_j^{net} - \sum_{i'<i} e^*_{i'j})$;

  if $v_i^{net} = \sum_{j'<j} e^*_{ij'}$ **then**

    $i = i + 1$;

  end

  if $v_j^{net} = \sum_{i'<i} e^*_{i'j}$ **then**

    $j = j + 1$;

  end

end

Algorithm 1: A perfectly efficient non-conservative compression algorithm with minimal density

From the initial market, the algorithm constructs two sets of nodes N^+ and N^- which contain nodes with positive and negative net positions, respectively. Note that nodes with 0 net positions (i.e., perfectly balanced position) will ultimately be isolated. They are thus kept aside from this point on. In addition, those two sets are sorted from the lowest to the highest absolute net position. The goal is then to generate a set of edges such that the resulting network is in line with the net position of each node. Starting from the nodes with the highest absolute net position, the algorithm generates edges in order to satisfy the net position of at least one node in the pair (i.e., the one with the smallest need). For example, if the node with highest net positive position is i with v_i^{net} and the node with lowest net negative position is j with v_j^{net}, an edge will be created such that the node with the lowest absolute net positions does not need more edges to satisfy its net position constraint. Assume that the nodes i and j are isolated nodes at the moment of decision, an edge $e_{ij} = min(v_i^{net}, v_j^{net})$ will thus be generated. In the more general case where i and j might already have some trades, we discount them in the edge generation process: $e^*_{ij} = \min(v_i^{net} - \sum_{j'<j} e^*_{ij'}, v_j^{net} - \sum_{i'<i} e^*_{i'j})$. The algorithm finishes once all the nodes have the net and gross positions equal.

The characteristics of the market resulting from a compression that follows the above algorithm are the following

Given a financial network G and a compression operator c() that is defined by the Algorithm 1, the resulting financial network $G_{min} = c(G)$ is defined as:

$$e_{ij} = \begin{cases} \min(v_n^i - \sum_{j'<j} e_{ij'}, v_n^j - \sum_{i'<i} e_{i'j}), & \text{if } v_n^i \cdot v_n^j < 0 \\ 0, & \text{otherwise} \end{cases}$$

where $i \in N^+ = \{s \text{ s.t. } v_n^s > 0\}$ and $j \in N^- = \{s \text{ s.t. } v_n^s < 0\}$.

Moreover:

- G_{min} is net-equivalent to G
- $\Delta(G_{min}) = 0$

E.2 Conservative algorithm

As we did for the non-conservative case, we now propose and analyze a conservative algorithm with the objective function of minimizing the excess of a given market with two constraints: (1)

keep the net positions constant and (2) the new set of trades is a subset of the previous one.

Data: Original Market G=(N,E)

Result: G^* such that $\Delta(G^*) < \Delta(G)$ and $e^*_{ij} < e_{ij}$

Let Π be set the of all directed closed chains in G;

Let $G^* = G$;

while $\Pi \neq \emptyset$ **do**

Select $P = (N', \mathbf{E}') \in \Pi$ such that

$|N'|.min_{e_{ij}}(\mathbf{E}) = max_{P_i=(N'_i,E'_i)\in\Pi}(|N'_{P_i}|.min_{e\in\mathbf{E}'_{P_i}}(e)))$;

$e_{ij} = e_{ij} - min_{e_{ij}}(\mathbf{E}')$ for all $e_{ij} \in \mathbf{E}'$;

$\mathbf{E}^* = \mathbf{E}^* \setminus \{e : e = min(\mathbf{E}')\}$;

$\Pi \setminus \{P\}$

end

Algorithm 2: A deterministic conservative compression algorithm

The algorithm works as follows. First, it stores all the closed chains present in the market. Then, it selects the cycle (i.e., closed chain) that will result in the maximum marginal compression (at the cycle level), that is, the cycle where the combination of the number of nodes and the value of the lowest trades is maximized. From that cycle, the algorithm removes the trade with the lowest notional and subtracts this value from the all the trades in the cycle. It then removes the cycle from the list of cycles and iterates the procedure until the set of cycles in the market is empty.

At each cycle step t of the algorithm, the excess of the market is reduced by:

$$\Delta_t(G') = \Delta_{t-1}(G') - |N'|min_{e_{ij}}(\mathbf{E}')$$

At the end of the algorithm, the resulting compressed market does not contain directed closed chains anymore: it is a Directed Acyclic Graph (DAG). Hence no further conservative compression can be applied to it.

F Efficiency ratios: invariance under scale transformations

We show that the both the excess ratio and the compression efficiency ratio for conservative compression are invariant to scale transformations.

Lemma 3. *Let $G = (N, \mathbf{E})$ a market with associated exposure matrix e_{ij}, and $G(\alpha) = (N, \mathbf{E}(\alpha))$ a market with exposure matrix $e_{ij}(\alpha) = \alpha \times e_{ij}$, where α is a strictly positive real number. The following relations hold:*

1. $v_i^{net}(\alpha) = \alpha v_i^{net} \;\; \forall i \in V$;
2. $x(\alpha) = \alpha x$, *where* $x = \sum_{ij} e_{ij}$ *and* $x(\alpha) = \sum_{ij} e_{ij}(\alpha)$;
3. $m(\alpha) = \alpha m$, *where m is the minimum total notional required to satisfy every participants' net position as defined in Eq 1 in Proposition 1 and used in Proposition 12.*
4. $\Delta(G(\alpha)) = \alpha \Delta(G)$;
5. $\epsilon(G(\alpha)) = \epsilon(G)$;
6. $\rho(G(\alpha)) = \rho(G)$.

Proof. Point 1 holds since

$$v_i^{\text{net}}(\alpha) = \sum_j \alpha e_{ij} - \sum_j \alpha e_{ji} = \alpha v_i^{\text{net}},$$

which implies that each net position is simply rescaled by a factor α. Points 2 and 3 are easily proven by multiplying by α and hence 4 and 5 follow straighforwardly by the definition of excess.

For point 6, we exploit the programming characterisation of the conservative compression problem and show that the optimal solutions of the program for $G(\alpha)$ coincides with that of G rescaled by α.

The program for $G(\alpha)$ can be expressed as follows:

$$\begin{array}{ll} \min & \frac{1}{\alpha} \sum_{ij} e'_{ij}(\alpha) \\ \text{s.t.} & \frac{1}{\alpha} \sum_j \left(e'_{ij}(\alpha) - e'_{ji}(\alpha) \right) = \frac{1}{\alpha} v_i(\alpha) = \frac{1}{\alpha} \alpha v_i^{\text{net}}, \forall i \in N \\ & 0 \leq \frac{1}{\alpha} e'_{ij}(\alpha) \leq \frac{1}{\alpha} e_{ij}(\alpha) \end{array}$$

By posing $e'_{ij}(\alpha) = \alpha e^{\star}_{ij}$ we observe that $e^{\star}_{ij} = e'_{ij}$. Point 6 follows by computing the ratio $\rho(G(\alpha))$ and applying 4.

■

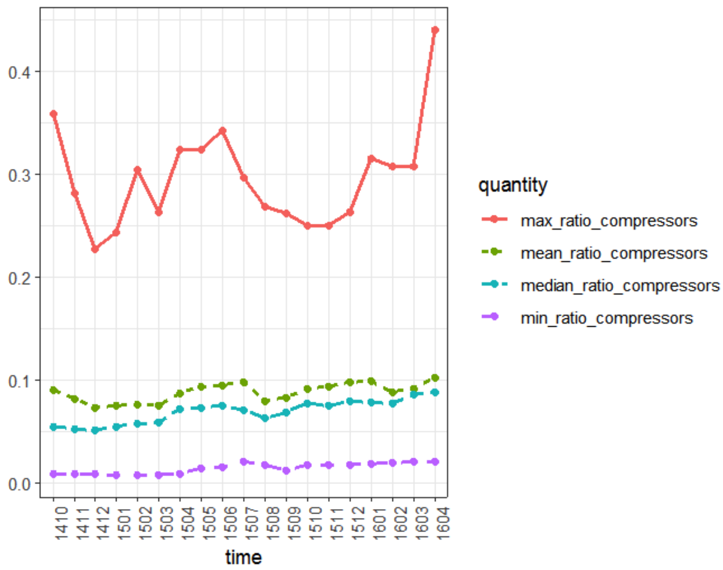


Figure 9: Statistics on the number of participants engaging in some form of portfolio compression.

G Compression adoption statistics

Figure 9 provides statistics on the share of participants engaging in portfolio compression in the markets we analyze. The count is made from flags in transaction reports which indicate whether a trade has been subject to compression. For each market and time step, we compute the number of participants who have at least one trade with the flag on. The figure shows that the number of participants engaging in portfolio compression is limited: on average less than 10% of participants report compressed trades while the markets where compression is used the most have a participation lower than 50%.

H General statistics

Table 7 reports the main statistics of the sampled data over time. The total notional of the selected 100 entities varies between 380Bn Euros and 480Bn Euros retaining roughly $30-34\%$ of the original total gross notional. The average number of counterparties across the 100 entities is stable and varies between 45 and 58 individual counterparties.

Time	Gross notional of 100 top ref. (E+11 euros)	Share of gross notional of 100 top ref.	Avg. participants per ref.
Oct-14	3.88	0.358	54
Nov-14	4.16	0.349	55
Dec-14	4.4	0.357	58
Jan-15	4.73	0.361	57
Feb-15	4.67	0.355	57
Mar-15	4.35	0.351	51
Apr-15	3.87	0.338	46
May-15	3.91	0.337	45
Jun-15	3.86	0.343	47
Jul-15	3.9	0.347	50
Aug-15	3.9	0.344	52
Sep-15	3.94	0.350	53
Oct-15	4.08	0.349	55
Nov-15	4.18	0.351	55
Dec-15	4.24	0.348	55
Jan-16	4.39	0.351	55
Feb-16	4.33	0.348	56
Mar-16	3.94	0.350	49
Apr-16	4.37	0.352	49

Table 7: General coverage statistics of the dataset over time: total outstanding gross notional of the sampled markets, share of sampled market's gross notional against the full dataset and average number of participant in each sampled market.

I Excess and efficiency in bilaterally compressed markets

In derivatives market like CDS markets, participants, specially dealers, reduce some positions by writing a symmetric contract in the opposite direction with the same counterparty. Analyzing the bilaterally compressed market thus allows us to quantify excess and compression efficiency beyond the redundancy incurred by this specific behavior.

As we have seen, bilateral excess, on average, accounts for half the excess of the original markets. In order to understand excess and compression beyond bilateral offsetting, we analyze further the bilaterally compressed markets. First, we obtain dealer-customer network characteristics reported in Table 8 after bilateral compression. While the participant-based statistics mirror Table 1, there is a reduction in all obligation-related statistics except the intra-customer density which remains the same: the average number of obligations is reduced by 25 percentage points. while the intra-dealer share of notional is only affected by 5 percentage points. Hence, we see that, despite the density reduction, the bulk of the activity remains in the intra-dealer activity after bilateral compression.[10]

In terms of excess, Table 9 complements the results from the bilateral compression efficiency and reports statistics similar to Table 2.[11] At the extremes, we note again high degrees of variability: for example, in mid-January 2016, the minimum level of excess was 0.261 while the maximum was 0.809. Nevertheless, results on the means and medians are stable over time and alway higher than 0.5. We thus see that, in general, around half of the gross notional of bilaterally compressed market remains in excess vis-a-vis market participants' net position. Note that the gross notional used here is the total notional left after bilateral compression on the original market.

Table 10 reports the results related to the efficiency of conservative and hybrid compression applied to the already bilaterally compressed market. On the extremes, both the conservative and the hybrid compression perform with various degrees of efficiency: the minimum amount of excess reduction via conservative compression (resp. hybrid compression) oscillates around 15% (resp. 35%) while the maximum amount of excess oscillates around 90% (resp. 97%). This shows that compression can perform very efficiently and very poorly with both approaches. However, the fact that conservative compression reaches 90% of excess removal shows the possibility of having very efficient compression despite restrictive compression tolerances. The mean and the median of both approaches are stable over time: both around 60% for the conservative compression and 75% for the hybrid compression. Overall, we find that each compression algorithm is able to remove more than half of the excess from the market. The hybrid compression allows for greater performances as a result of relaxing intra-dealer compression tolerances.

[10]Note that the average intra-customer density is equal to Table 1. In theory, we should have doubled the value as the density of the bilaterally netted intra-customer segment should be seen as the density of a undirected graph. We kept the previous definition to highlight the fact that the intra-customers obligations are not affected by the bilateral compression and avoid a misinterpretation of density increase.

[11]The relationship between the bilateral compression efficiency, ρ_b and the relative excesses in the original market, ϵ^o, and the bilaterally compressed market, ϵ^b, is given by $\rho_b = (1 - \frac{1-\epsilon^o}{1-\epsilon^b})\frac{1}{\epsilon^o}$. This expression directly follows from the definition of each parameter.

Time	Avg. num. dealers	Avg. num. customers buying	Avg. num. customers selling	Avg. num. obligations	Avg. share intra dealer notional	Avg. density	Avg. intr dealer density	Avg. intra customer density
Oct-14	18	16	20	115.87	0.767	0.075	0.221	0.0010
Nov-14	19	16	21	121.61	0.779	0.077	0.227	0.0006
Dec-14	19	17	21	128.24	0.777	0.076	0.221	0.0005
Jan-15	19	17	21	127.94	0.778	0.075	0.219	0.0006
Feb-15	19	17	21	126.31	0.782	0.075	0.221	0.0004
Mar-15	18	15	17	114.74	0.786	0.078	0.225	0.0007
Apr-15	18	13	15	106.28	0.786	0.079	0.229	0.0005
May-15	18	12	15	106.18	0.782	0.078	0.224	0.0008
Jun-15	18	13	14	105.20	0.783	0.076	0.216	0.0010
Jul-15	19	14	14	107.05	0.766	0.072	0.211	0.0009
Aug-15	19	15	17	111.49	0.776	0.073	0.208	0.0011
Sep-15	19	16	17	114.12	0.755	0.071	0.204	0.0018
Oct-15	19	16	18	117.22	0.766	0.072	0.200	0.0013
Nov-15	19	17	19	120.52	0.762	0.072	0.198	0.0017
Dec-15	19	17	19	120.76	0.772	0.072	0.198	0.0012
Jan-16	20	17	18	121.06	0.774	0.072	0.198	0.0013
Feb-16	19	17	18	121.05	0.763	0.071	0.197	0.0012
Mar-16	18	14	17	108.03	0.739	0.070	0.205	0.0018
Apr-16	19	14	17	109.29	0.759	0.071	0.204	0.0019

Table 8: Dealers/customers statistics after bilateral compression.

Total Excess	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.422	0.423	0.290	0.257	0.366	0.261	0.293
max	0.811	0.811	0.798	0.809	0.820	0.809	0.781
mean	0.614	0.621	0.614	0.602	0.597	0.570	0.558
stdev	0.087	0.087	0.091	0.095	0.097	0.112	0.098
first quart.	0.562	0.558	0.562	0.544	0.531	0.489	0.503
median	0.617	0.618	0.614	0.613	0.594	0.569	0.566
third quart.	0.670	0.684	0.674	0.663	0.654	0.653	0.635

Table 9: Excess statistics after bilateral compression

Conservative (ρ_c)	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.160	0.203	0.140	0.163	0.165	0.119	0.098
max	0.894	0.927	0.923	0.878	0.912	0.911	0.878
mean	0.568	0.622	0.599	0.592	0.555	0.552	0.525
stdev	0.166	0.160	0.164	0.158	0.175	0.183	0.172
first quart.	0.456	0.505	0.512	0.489	0.435	0.437	0.409
median	0.562	0.636	0.594	0.591	0.537	0.550	0.546
third quart.	0.685	0.729	0.728	0.705	0.680	0.687	0.643

Hybrid (ρ_h)	Oct-14	Jan-15	Apr-15	Jul-15	Oct-15	Jan-16	Apr-16
min	0.370	0.460	0.377	0.281	0.259	0.281	0.135
max	0.971	0.973	0.968	0.963	0.977	0.974	0.981
mean	0.724	0.763	0.760	0.755	0.738	0.735	0.752
stdev	0.149	0.130	0.130	0.130	0.146	0.140	0.148
first quart.	0.623	0.691	0.678	0.674	0.626	0.642	0.679
median	0.735	0.785	0.781	0.778	0.775	0.756	0.784
third quart.	0.846	0.866	0.859	0.866	0.849	0.851	0.845

Table 10: Statistics of compression efficiency after bilateral compression